Contextual Bandit Algorithms with Supervised Learning Guarantees

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Serving Content to Users

Query, IP address, browser properties, etc.

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result (ie. ad, news story)

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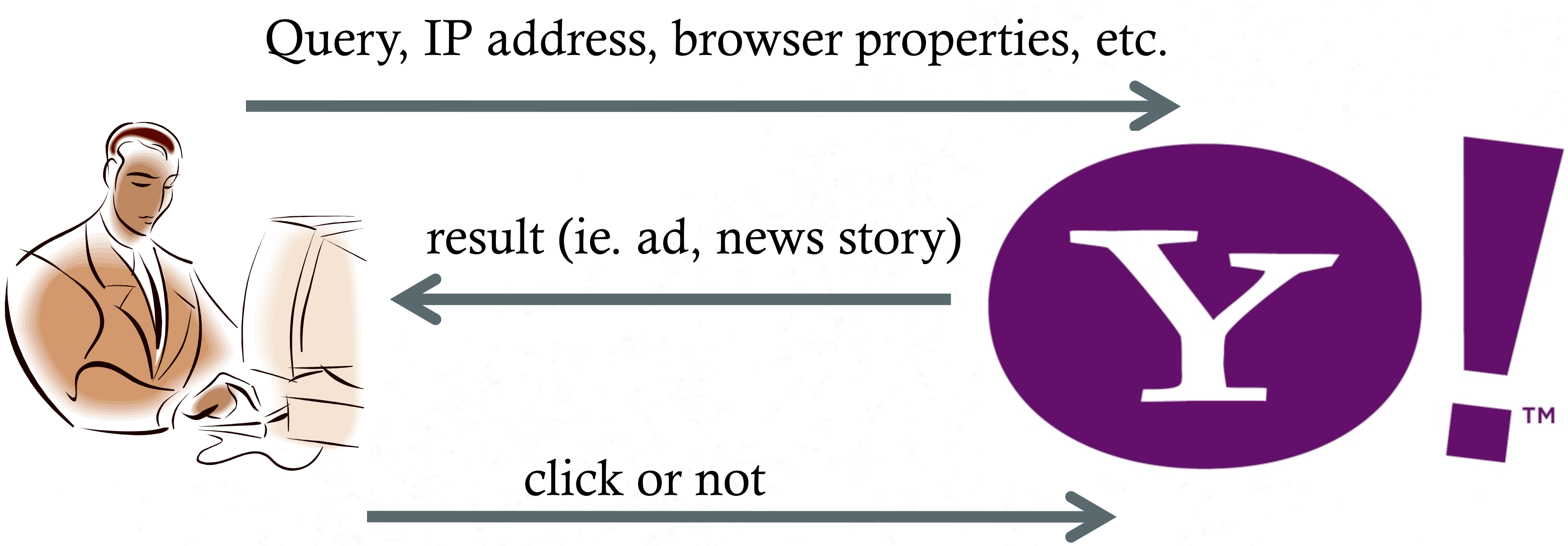

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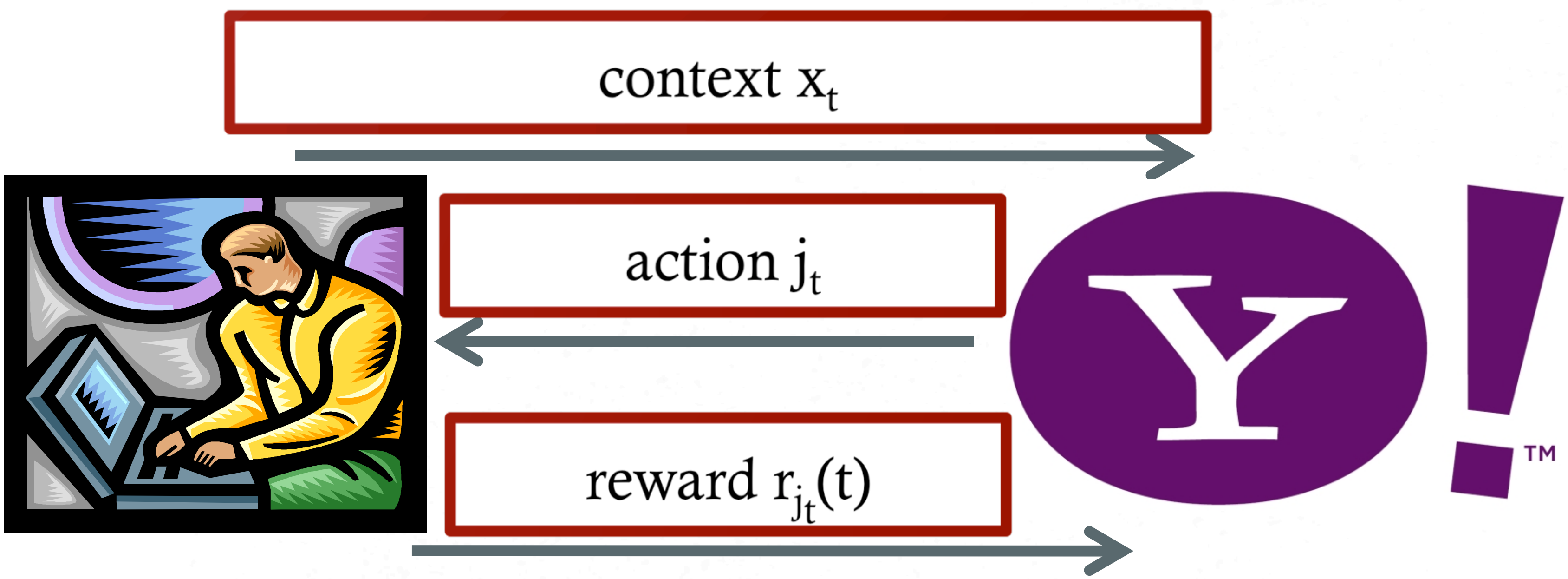

Outline

- **Formally define the setting.**
- Show ideas that fail.
- Give a high probability optimal algorithm.
- Dealing with VC sets.
- Experiments

The Contextual Bandit Setting

- T rounds, K possible actions, N policies π in Π (context → actions)
- for t=1 to T
  - world commits to rewards **r(t)**=$r_1(t), r_2(t), \ldots, r_K(t)$
  - world provides context x_t
  - learner's policies recommend $\pi_1(x_t)$, $\pi_2(x_t)$, …, $\pi_N(x_t)$
  - learner chooses action j_t
  - learner receives reward $r_{j_t}(t)$
- want to compete with following the best policy in hindsight

Regret

- reward of algorithm A: $G_A(T) \doteq \sum_{t=1}^{T} r_{j_t}(t)$
- expected reward of policy i: $G_i(T) \doteq \sum_{t=1}^{T} \pi_i(x_t) \cdot r(t)$
- algorithm A's regret: $\max_i G_i(T) - G_A(T)$

Regret

- algorithm A's regret: $\max_i G_i(T) - G_A(T)$
- expected regret: $\max_i G_i(T) - E[G_A(T)]$
- high probability regret: $P[\max_i G_i(T) - G_A(T) > \varepsilon] \leq \delta$

Some Observations

- This is harder than supervised learning. In the bandit setting we do not know the rewards of actions not taken.
- This is not the traditional K-armed bandit setting. In the traditional bandit setting there is no context (or experts).
  - In the simpler K-armed bandit setting, there is no context. We just compete with best arm in hindsight.
  - The traditional setting is akin to showing everyone the same advertisement, article, etc.

Previous Results

Algorithm	Regret	High Prob?	Contextual?
Exp4 [ACFS '02]	$\tilde{O}(KT \ln(N))^{1/2}$	No	Yes
ε -greedy, epoch-geedy [LZ '07]	$\tilde{O}((K \ln(N)^{1/3})T^{2/3})$	Yes	Yes
Exp3.P[ACFS '02] UCB [Auer '00]	$\tilde{O}(KT)^{1/2}$	Yes	No

$\Omega\left(\sqrt{KT}\right)$ lower bound [ACFS '02]

Our Result

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First Some Failed Approaches

- **Bad idea 1:** Maintain a set of plausible hypotheses and randomize uniformly over their predicted actions.

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- **Bad idea 2:** Maintain a set of plausible hypotheses and randomize uniformly among the hypothesis.
  - Adversary has two actions, one always paying off 1 and the other 0. If all but one of > 2T hypothesis always predict wrong arm, and only 1 hypothesis always predicts good arm, with probability > ½ it is never picked and algorithm incurs regret of T.

epsilon-greedy

- Rough idea of ε-greedy (or epoch-greedy [Langford and Zhang '07]): act randomly for ε rounds, otherwise go with best action (or policy).
- Even if we know the number of rounds in advance, epsilon-first won't get us regret $O(T)^{1/2}$, even in the non-contextual setting.
- Rough analysis: even for just 2 arms, we suffer regret: $\varepsilon + (T-\varepsilon)/(\varepsilon^{1/2})$.
  - $\varepsilon \approx T^{2/3}$ is optimal tradeoff.
  - gives regret $\approx T^{2/3}$
  - in comparison, in this paper we achieve $\approx T^{1/2}$

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Ideas Behind Exp4.P

- exponential weights
  - keep a weight on each expert that drops exponentially in the expert's (estimated) performance
- upper confidence bounds
  - use an upper confidence bound on each expert's estimated reward
- ensuring exploration
  - make sure each action is taken with some minimum probability
- importance weighting
  - give rare events more importance to keep estimates unbiased

Exponential Weight Algorithm for Exploration and Exploitation with Experts

(EXP4) [Auer et al. '95]

(slide from Beygelzimer & Langford ICML 2010 tutorial)

Initialization: $\forall \pi \in \Pi : w_t(\pi) = 1$

For each $t = 1, 2, \ldots$:

1. Observe x_t and let for $a = 1, \ldots, K$

$$p_t(a) = (1 - Kp_{\min}) \frac{\sum_\pi \mathbf{1}[\pi(x_t) = a]\, w_t(\pi)}{\sum_\pi w_t(\pi)} + p_{\min},$$

   where $p_{\min} = \sqrt{\frac{\ln |\Pi|}{KT}}$.

2. Draw a_t from p_t, and observe reward $r_t(a_t)$.
3. Update for each $\pi \in \Pi$

$$w_{t+1}(\pi) = \begin{cases} w_t(\pi) \exp\left(p_{\min} \frac{r_t(a_t)}{p_t(a_t)}\right) & \text{if } \pi(x_t) = a_t \\ w_t(\pi) & \text{otherwise} \end{cases}$$

Exponential Weight Algorithm for Exploration and Exploitation with Experts

(Exp4.P) [Beygelzimer, Langford, Li, R, Schapire '10]

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$\hat{y}_i(t)$ $\hat{v}_i(t)$

$$w_{t+1}(\pi) = w_t(\pi) \exp\left(\frac{p_{\min}}{2}\left(\mathbf{1}[\pi(x_t) = a_t]\frac{r_t(a_t)}{p_t(a_t)} + \frac{1}{p_t(\pi(x_t))}\sqrt{\frac{\ln N/\delta}{KT}}\right)\right)$$

Lemma 1

The estimated reward of an expert is $\hat{G}_i \doteq \sum_{t=1}^{T} \hat{y}_i(t).$

We also define $\hat{\sigma}_i \doteq \sqrt{KT} + \frac{1}{\sqrt{KT}} \sum_{t=1}^{T} \hat{v}_i(t).$

Lemma $\mathbf{Pr}\left[\exists i : G_i \geq \hat{G}_i + \sqrt{\ln(N/\delta)}\hat{\sigma}_i\right] \leq \delta.$

Proof uses a new Freedman-style martingale inequality.

Lemma 2

$$\hat{U} = \max_i \left(\hat{G}_i + \hat{\sigma}_i \cdot \sqrt{\ln(N/\delta)} \right).$$

Lemma

$$G_{\text{Exp4.P}} \geq \left(1 - 2\sqrt{\frac{K \ln N}{T}}\right) \hat{U} - 2\sqrt{KT \ln(N/\delta)} - \sqrt{KT \ln N} - \ln(N/\delta).$$

Proof tracks the weights of experts, similar to Exp4.

Lemmas 1 and 2 imply : $G_{\text{Exp4.P}} \geq G_{\max} - 6\sqrt{KT \ln(N/\delta)}.$

One Problem…

- This algorithm requires keeping explicit weights on the policies.
  - Okay for polynomially many policies.
  - Okay for some special cases.
  - Not efficient in general.
- Want an efficient algorithm that would (for example) work with an ERM Oracle
  - epoch-greedy [Langford and Zhang ’07] has this property.

Results

Algorithm	Regret	H.P.?	Context?	Efficient?
Exp4 [ACFS '02]	$\tilde{O}(T)^{1/2}$	No	Yes	No
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Infinitely Many Policies

- What if we have an infinite number of policies?
- Our bound of $\tilde{O}(K \ln(N) T)^{1/2}$ becomes vacuous.
- If we assume our policy class has a finite VC dimension d, then we can tackle this problem.
- Need i.i.d. assumption. We will also assume k=2 to illustrate the argument.

VC Dimension

- The VC dimension of a hypothesis class captures the class's expressive power.
- It is the cardinality of the largest set (in our case, of contexts) the class can shatter.
  - Shatter means to label in all possible configurations.

VE, an Algorithm for VC Sets

The VE algorithm:

- Act uniformly at random for τ rounds.
- This partitions our policies Π into equivalence classes according to their labelings of the first τ examples.
- Pick one representative from each equivalence class to make Π'.
- Run Exp4.P on Π'.

Outline of Analysis of VE

- Sauer's lemma bounds the number of equivalence classes to $(e\tau/d)^d$.
  - Hence, using Exp4.P bounds, VE's regret to Π' is ≈ τ + O(Td ln(τ))
- We can show that the regret of Π' to Π is ≈ (T/ τ)(dlnT)
  - by looking at the probability of disagreeing on future data given agreement for τ steps.
- $\tau \approx (Td \ln 1/\delta)^{1/2}$ achieves the optimal trade-off.
- Gives $\tilde{O}(Td)^{1/2}$ regret.
- Still inefficient!

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- **Experiments**

Experiments on Yahoo! Data

- We chose a policy class for which we could efficiently keep track of the weights.
  - Created 5 clusters, with users (at each time step) getting features based on their distances to clusters.
  - Policies mapped clusters to article (action) choices.
  - Ran on personalized news article recommendations for Yahoo! front page.

- We used a learning bucket on which we ran the algorithms and a deployment bucket on which we ran the greedy (best) learned policy.

Experimental Results

- Reported normalized estimated click-through-rates (rewards). Over 41M visits, with 253 articles and 21 candidate articles per visit.

	Exp4.P	Exp4	ε -greedy
learning eCTR	1.0525	1.0988	1.3829
deployment eCTR	**1.6512**	1.5309	1.4290

Summary

- Described Exp4P, the first optimal high probability algorithm for the contextual bandit problem.
- Showed how to compete with a VC-Set.
- Experimental Evidence for Exp4.Ps effectiveness.
- Main drawback is efficiency. We only have efficient algorithms for restricted classes, eg. our experiments, linear bandits (Auer 2002, Chu Li R Schapire 2011), etc.
- Main Open Problem: Find an **efficient** optimal algorithm for the contextual bandits problem!
  - Check out John Langford's talk at Snowbird!