Optimally Learning Social Networks with Activations and Suppressions

ALT 2008

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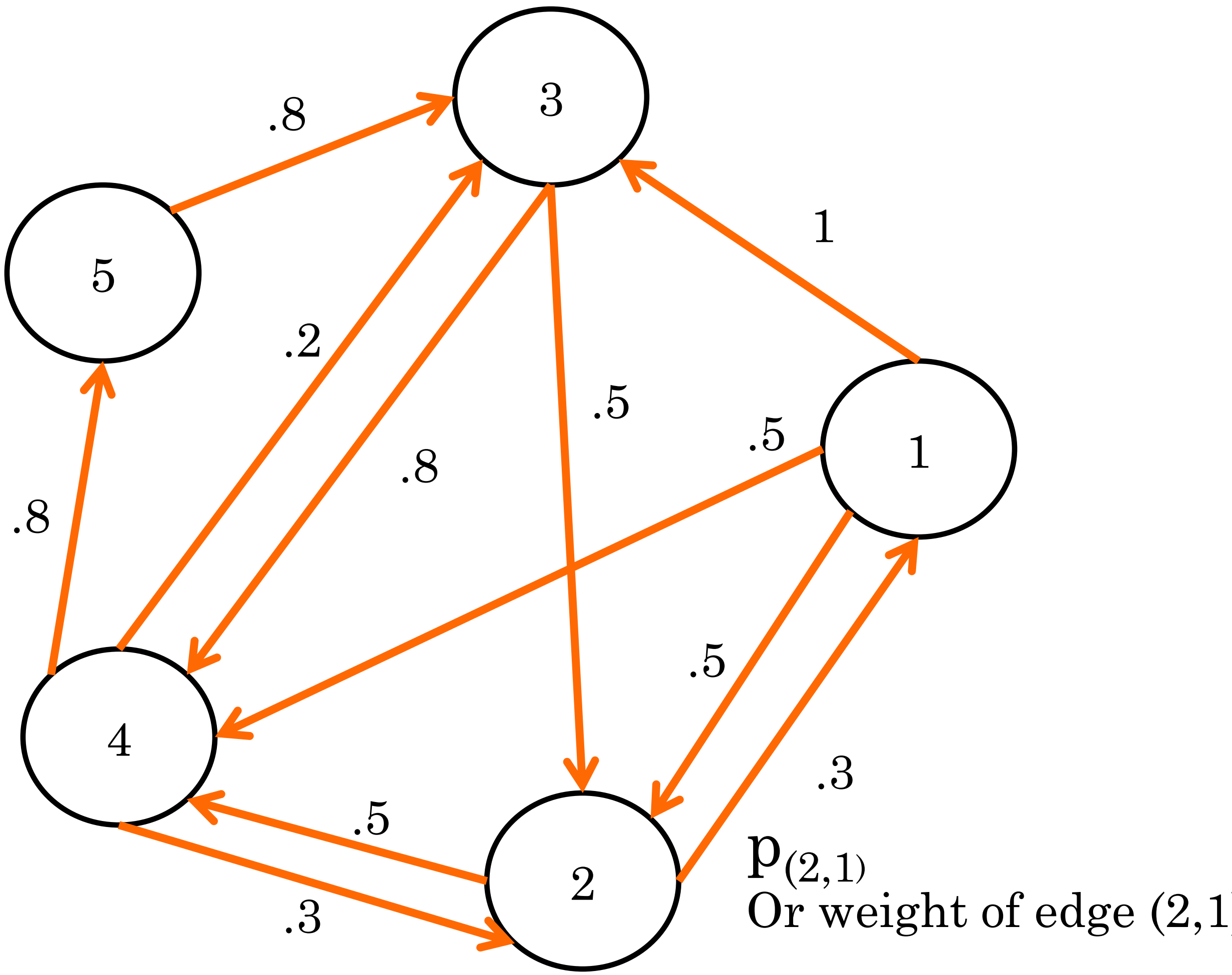
Talk Outline

- Independent Cascade Social Networks (SNs)
- Introduce Value Injection Queries (VIQs) on SNs
  - VIQs override the values of node (or gates in the case of circuits)
  - These were introduced by AACW (STOC '06), extended in AACR (COLT '07) and AACER (COLT '08)
- Show lower bound for learning SNs with VIQs
- Give an idea of the algorithm for learning SNs
- Other results
  - Approximation algorithm
  - Special case of trees
  - Exponential attenuation of paths

Independent Cascade Social Networks



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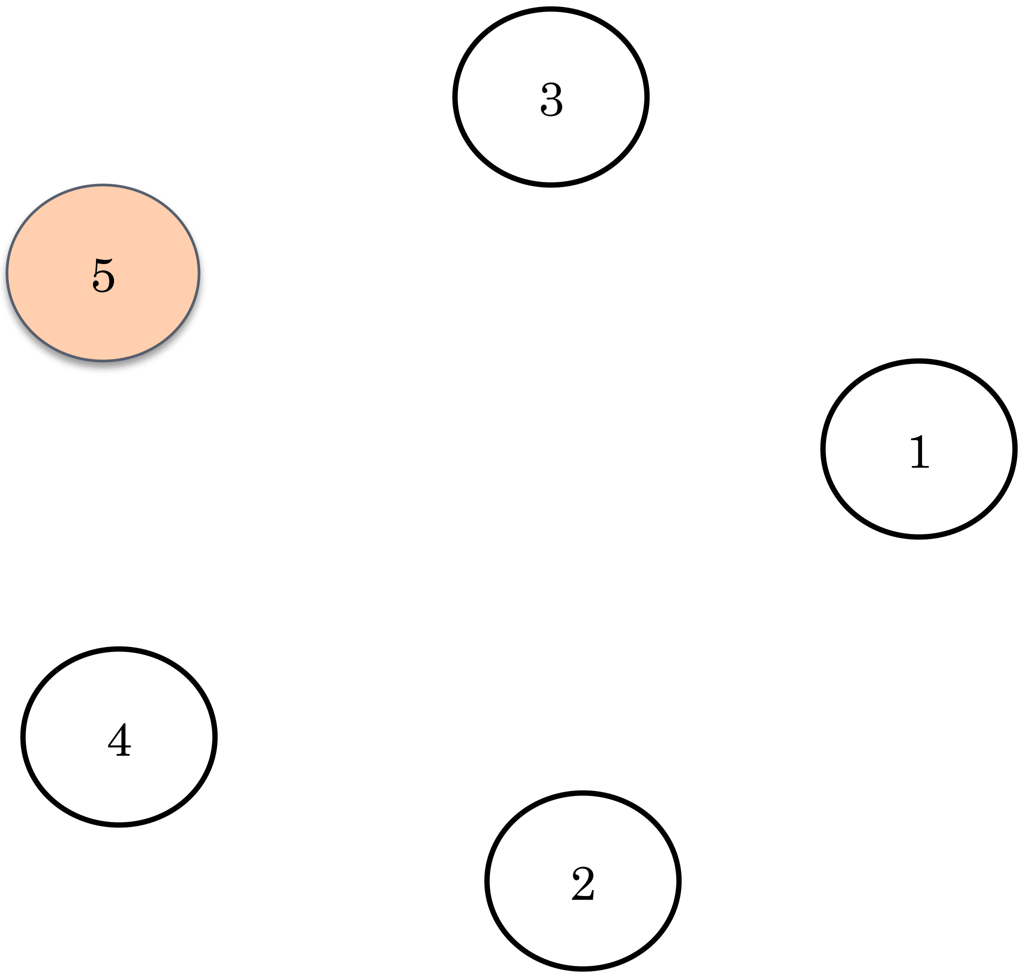
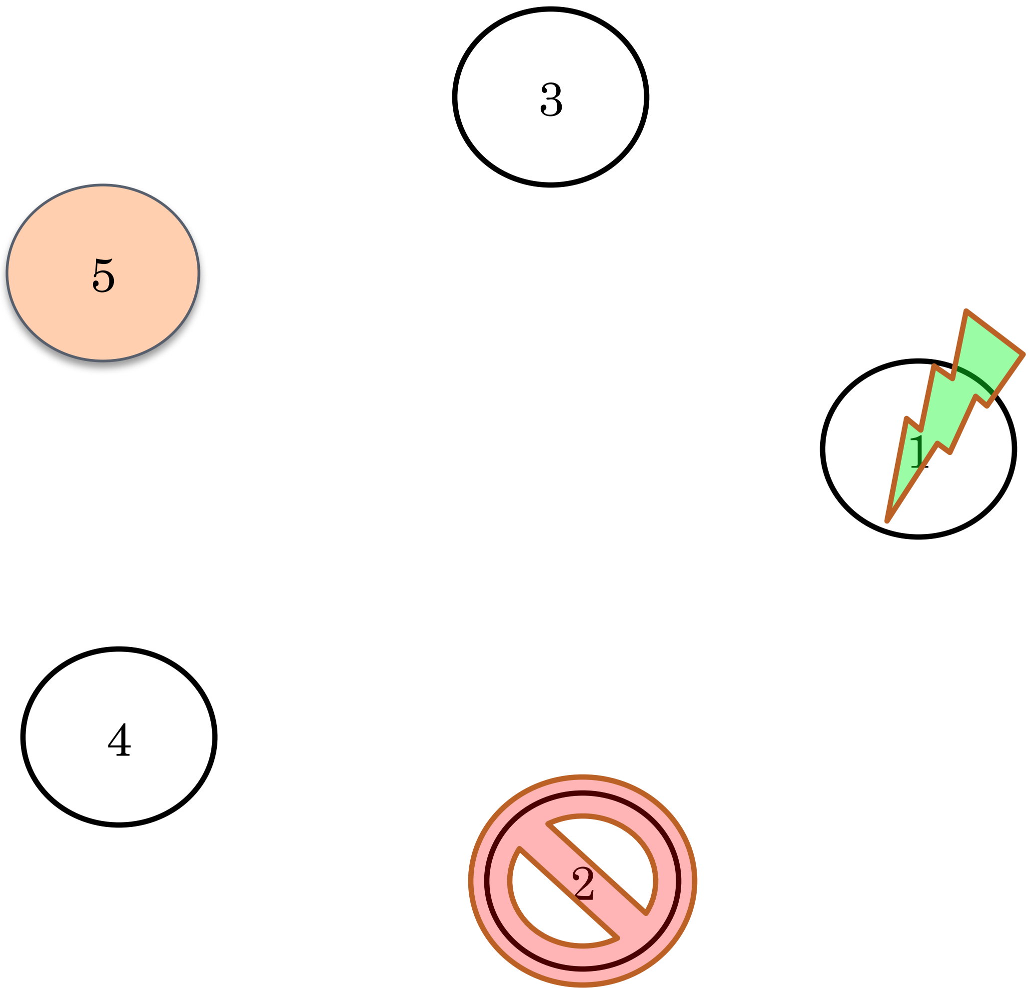
What the Learner Sees



Activations and Suppressions



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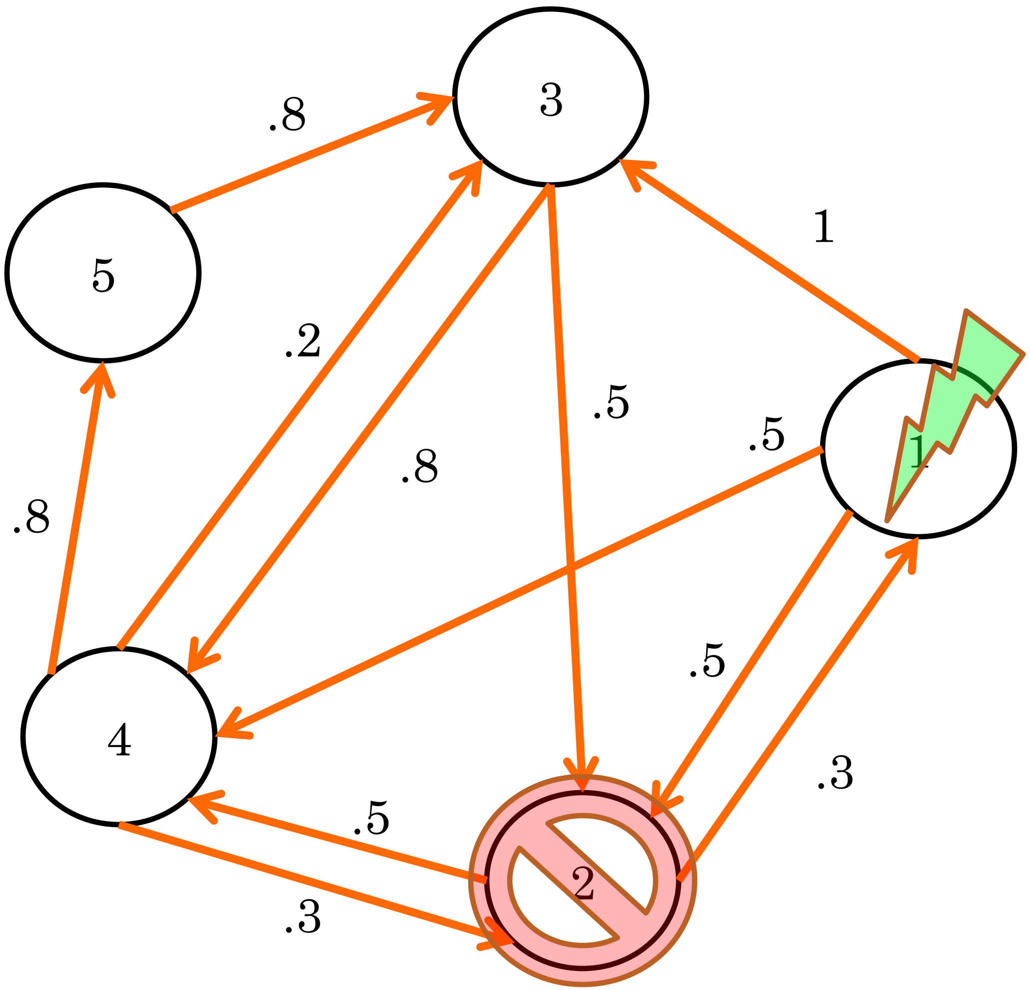

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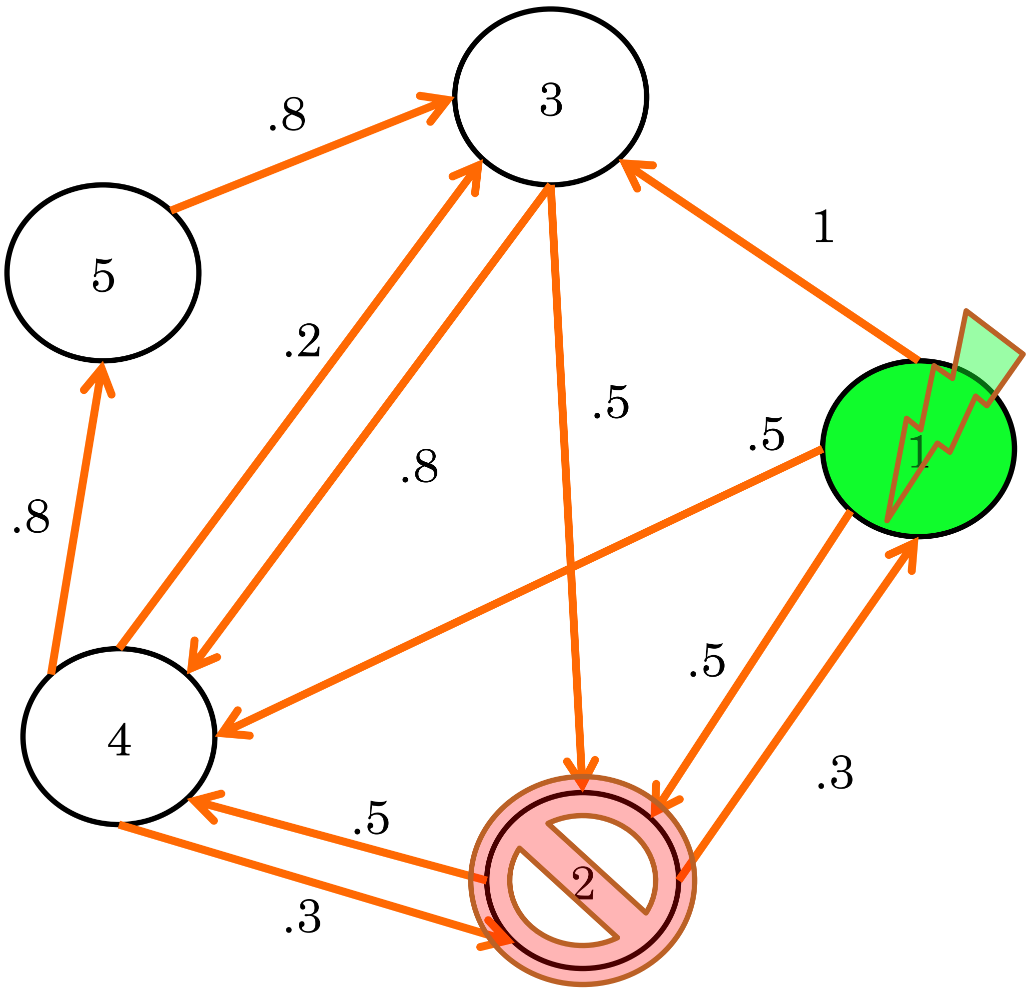

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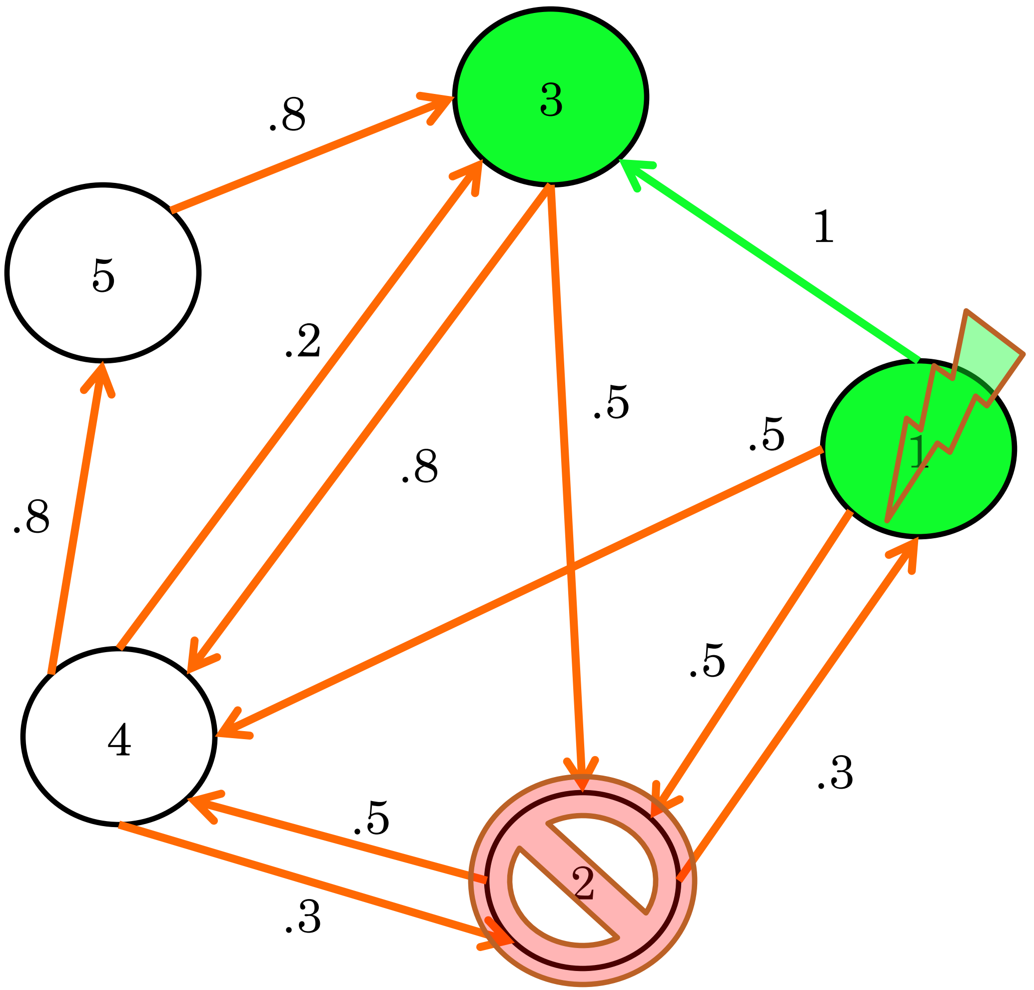

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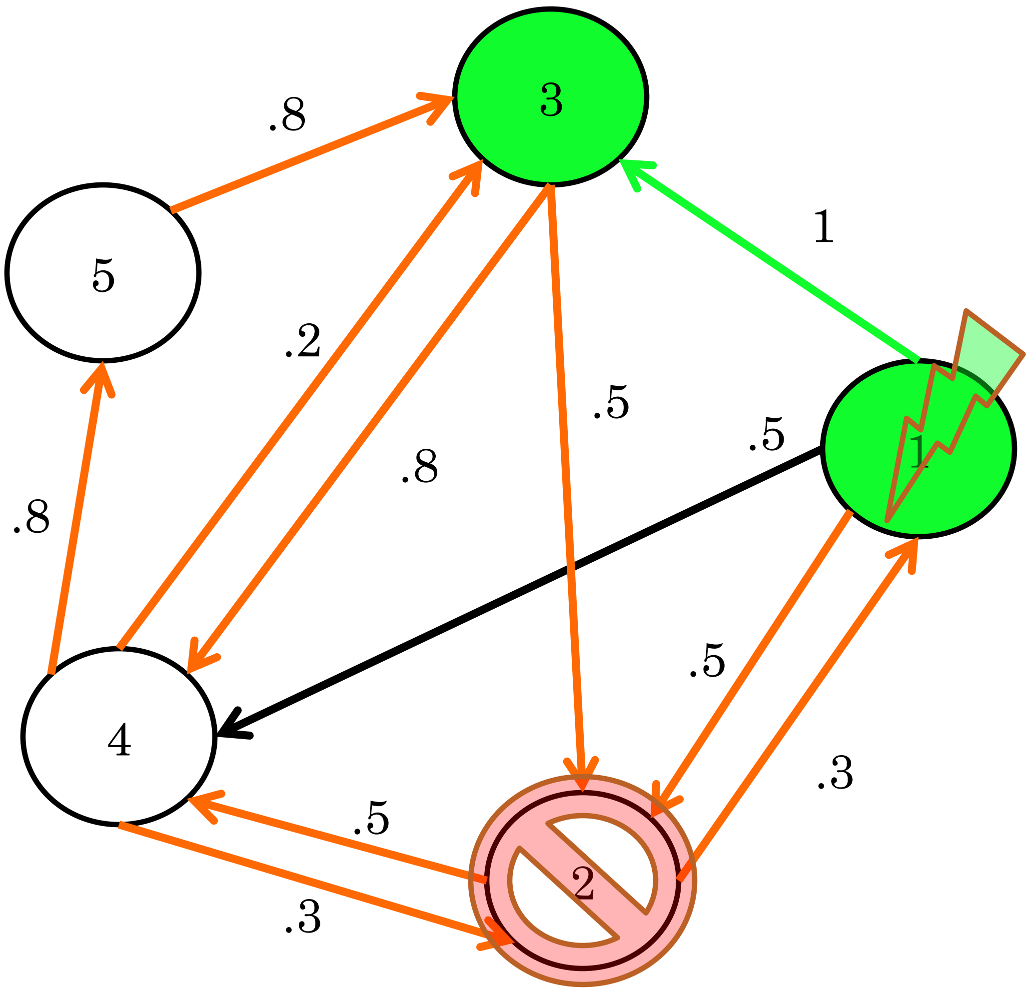

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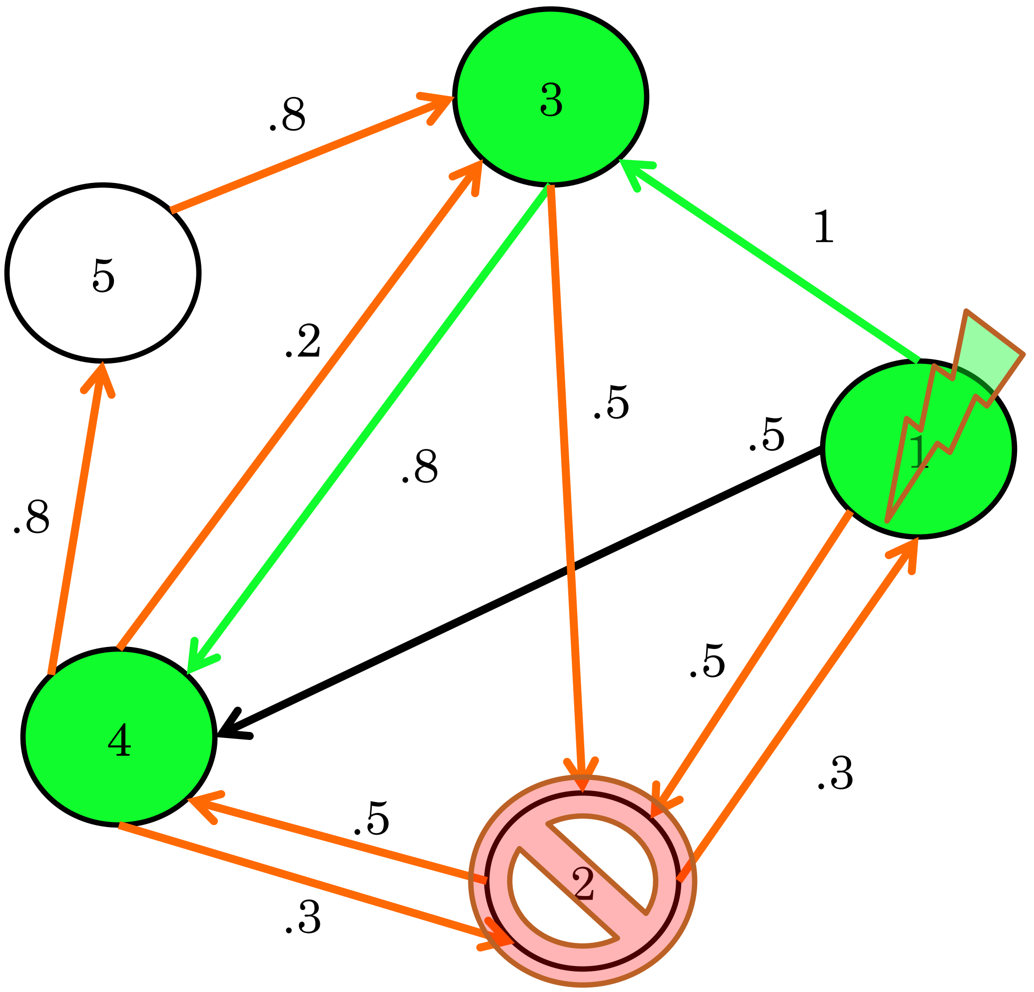

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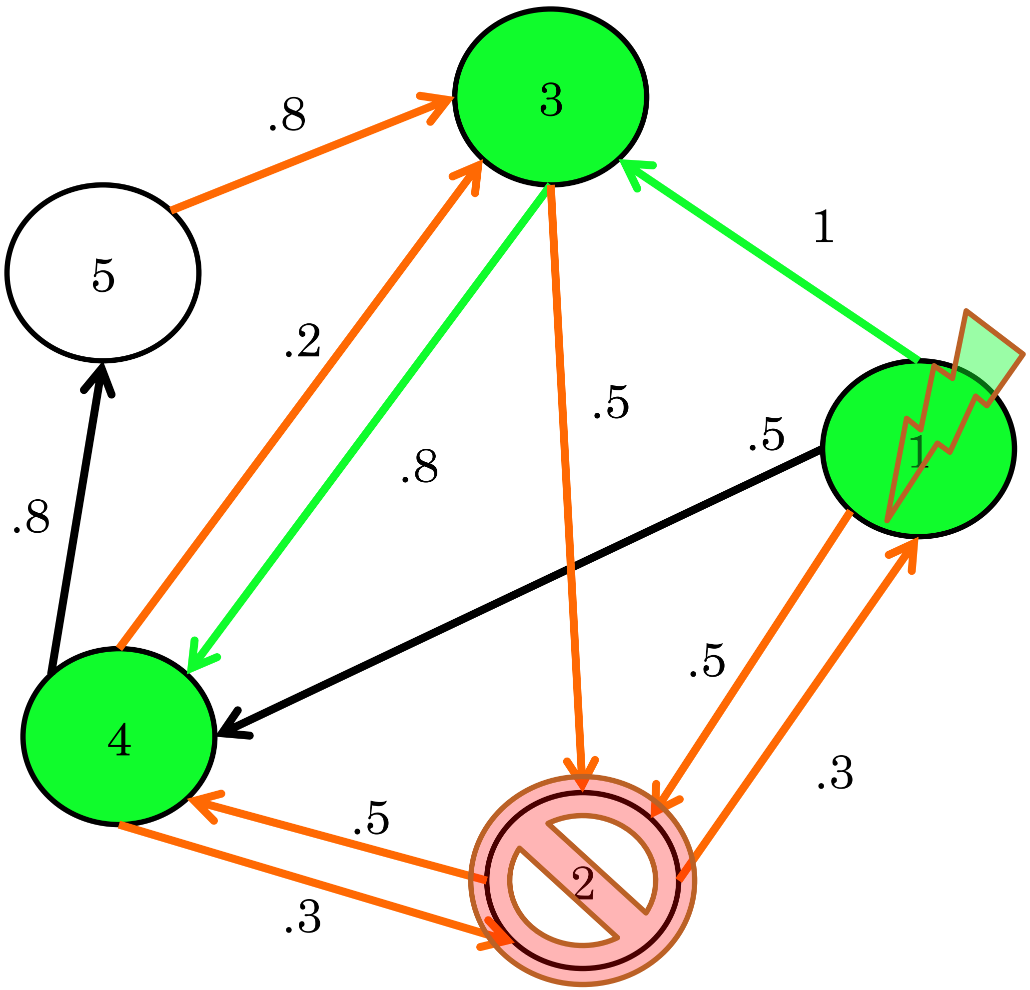

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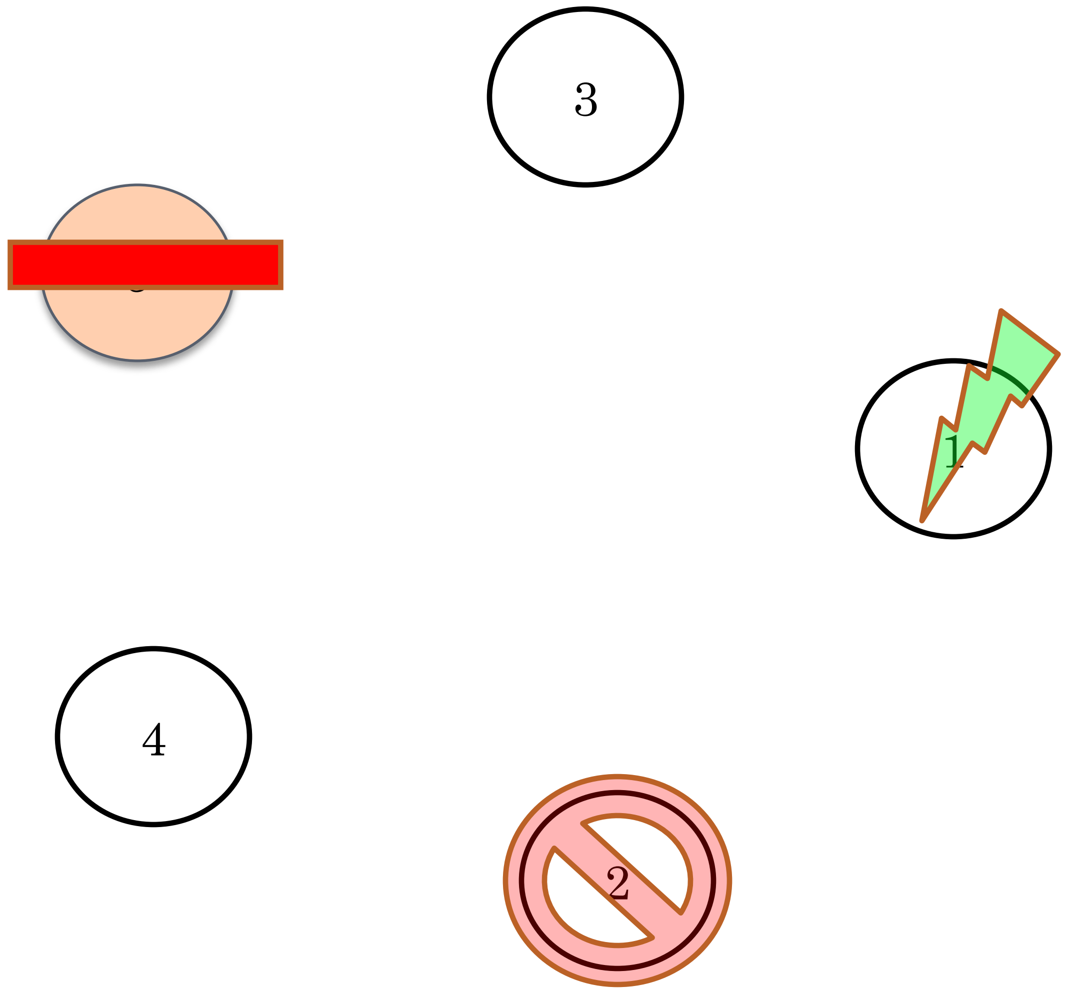

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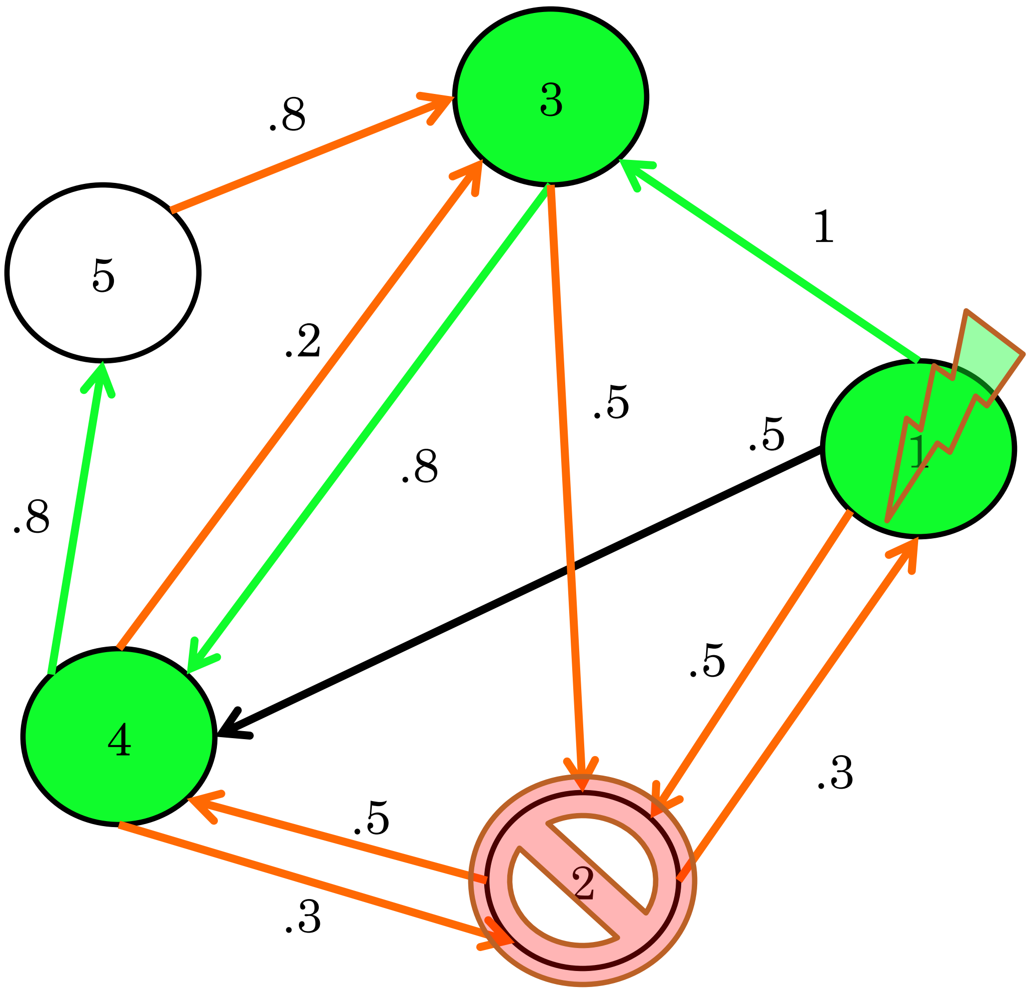

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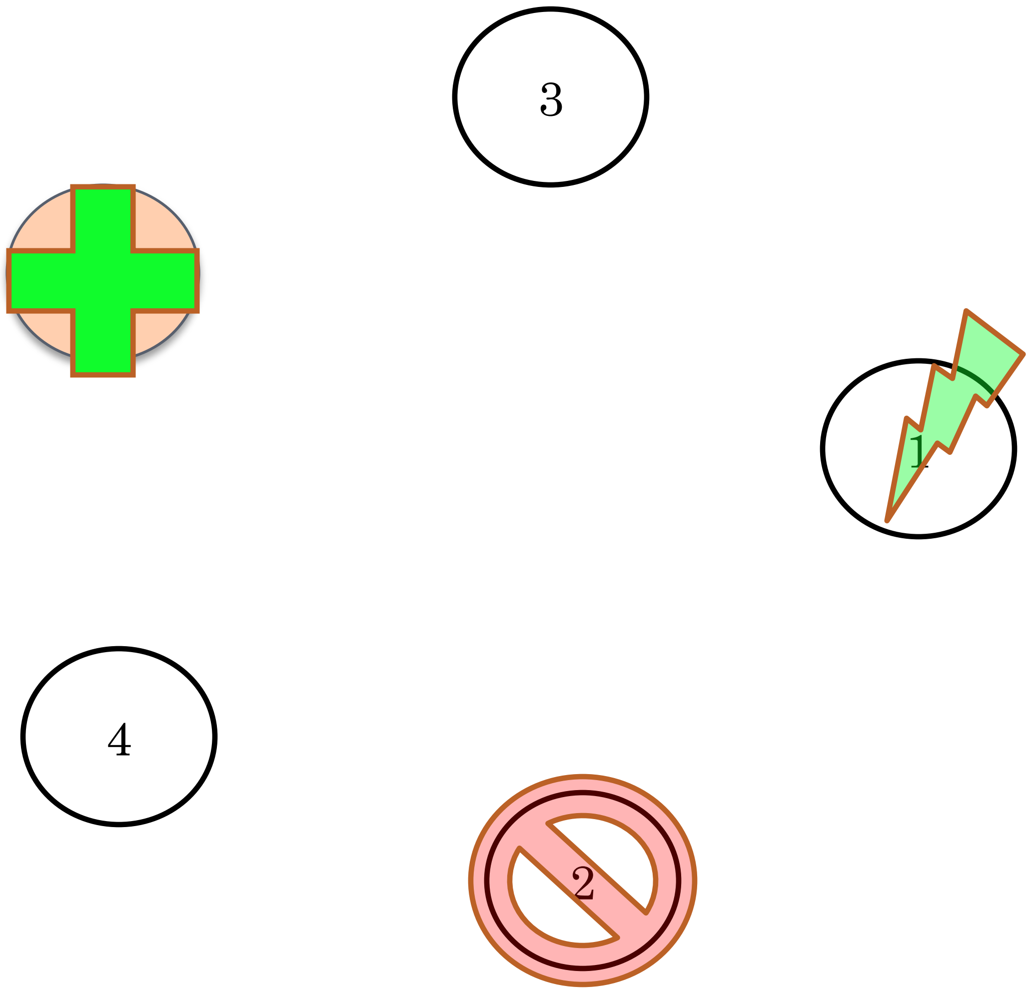
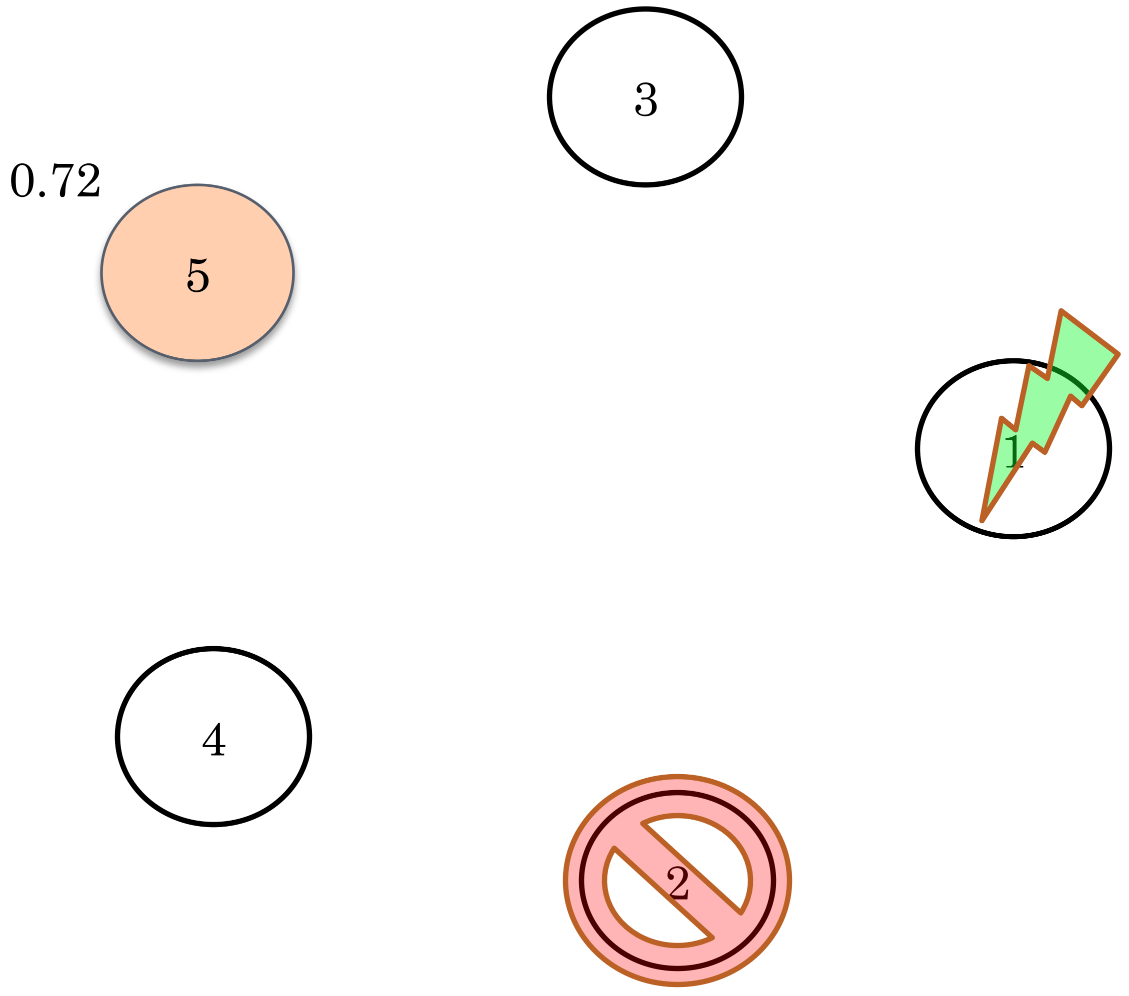


Exact Value Injection Queries



The Learning Task

- Two social networks S and S' are behaviorally equivalent if for any experiment e, S(e) = S'(e)
- Give access to a hidden social network S*, the learning problem is to find a social network S behaviorally equivalent to S* using value injection queries.

The Percolation Model

Given a network S and a VIQ

- All edges entering or leaving a suppressed node are automatically "closed."
- Each remaining edge (u,v) is "open" with probability $p_{(u,v)}$ and "closed" with probability (1- $p_{(u,v)}$)
- The result of a VIQ is the probability there is a path from a fired node to the output via open edges in S

Discoverable Edges

- Let S be a social network and S' be another social network that differs from S only in edge (u,v).
- We say edge (u,v) is discoverable if there is an experiment e such that $S(e) \neq S'(e)$.
- We can view the learning problem as having to find all discoverable edges.

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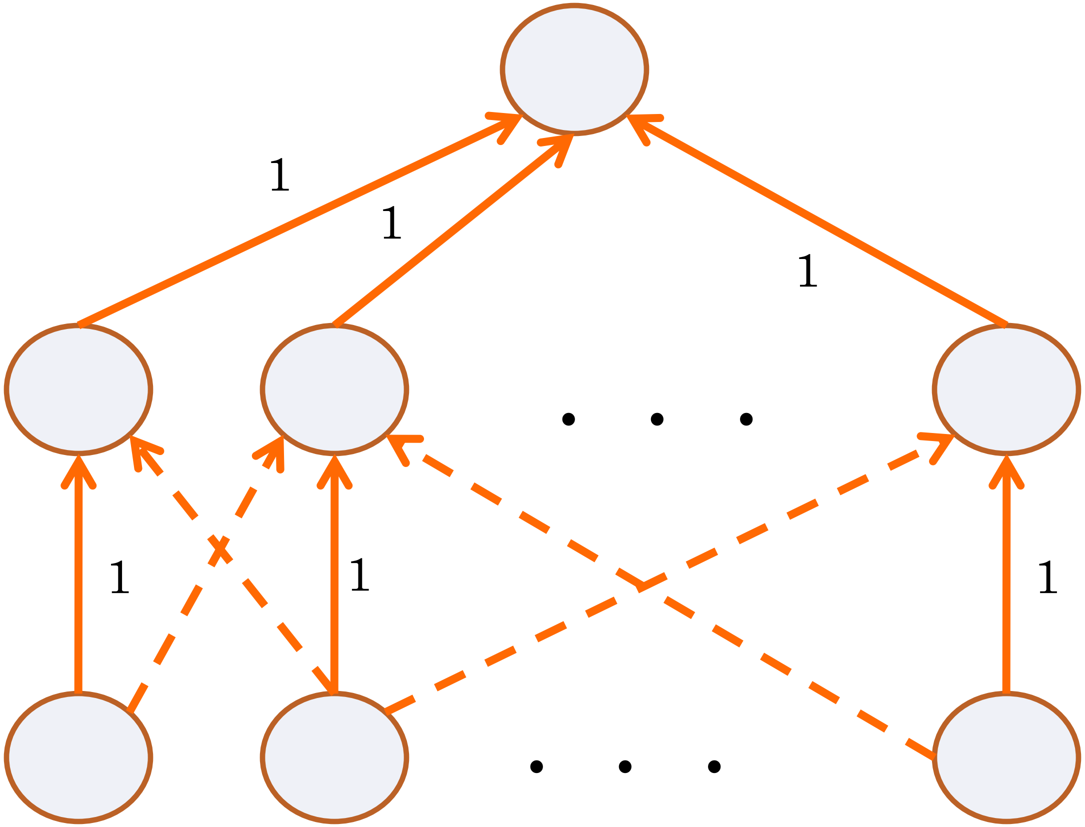
A Lower Bound



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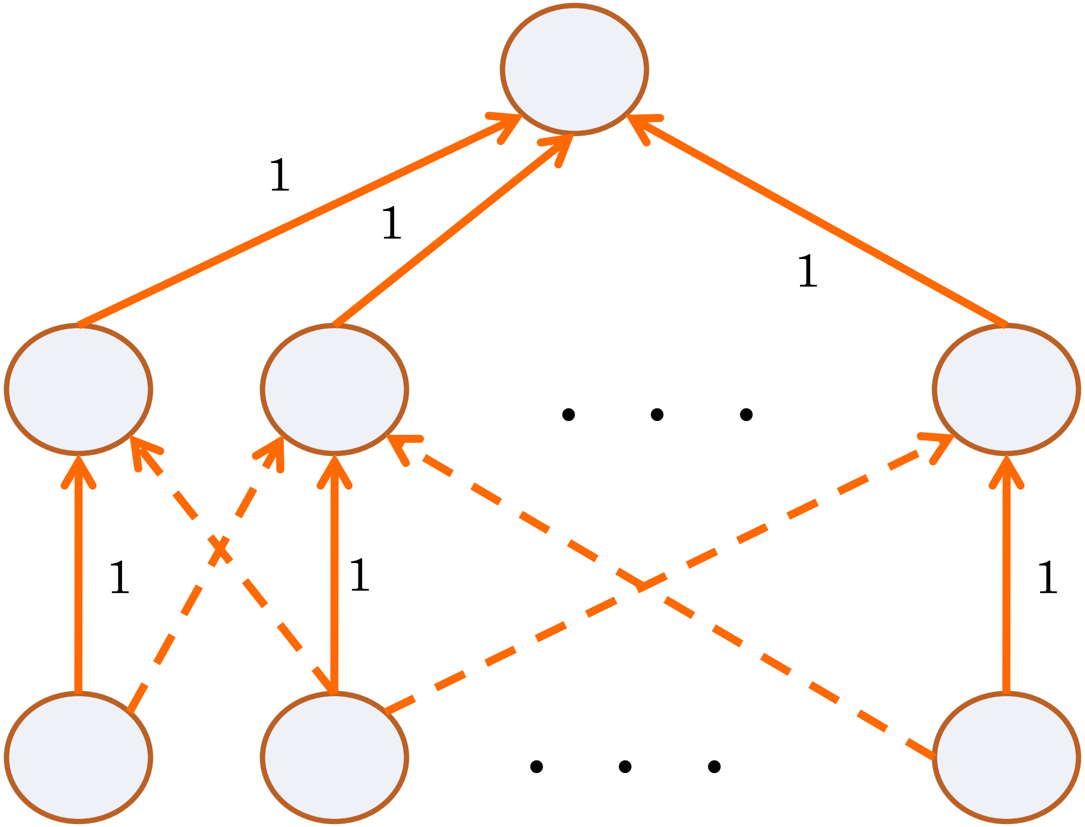
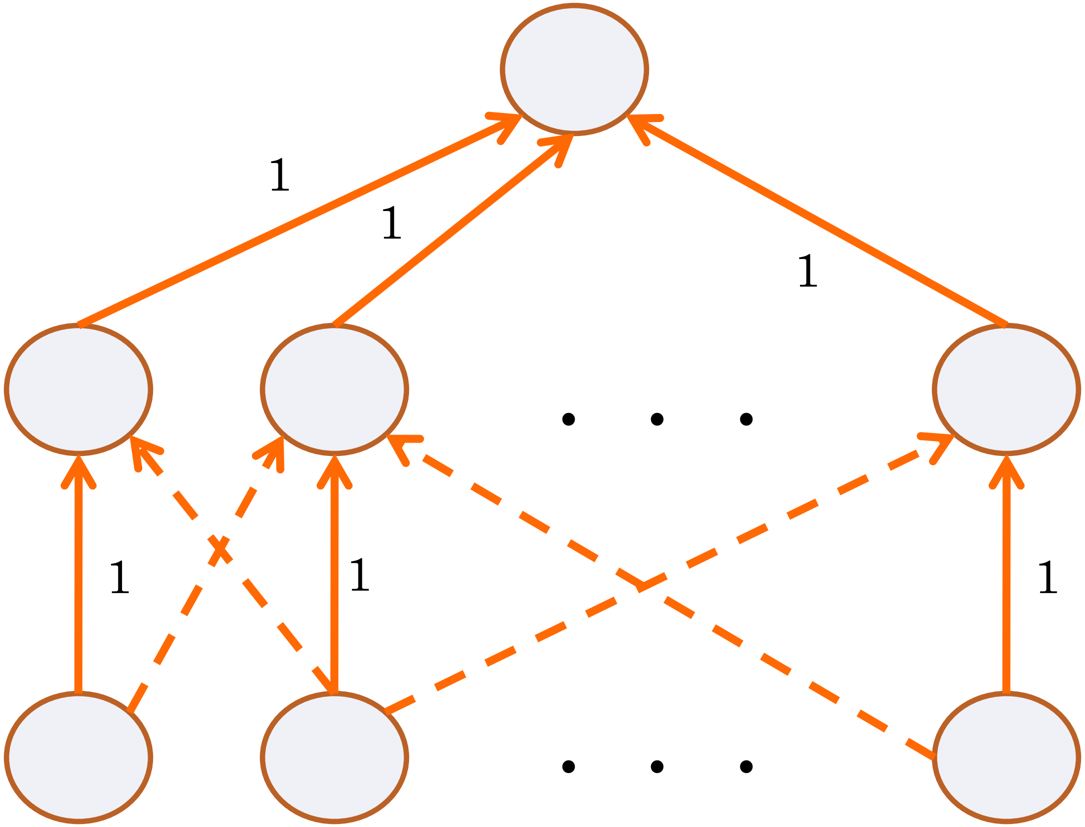


All queries give 1-bit answers

A Lower Bound



$2^{\Omega(n^2)}$ such graphs, $\Omega(n^2)$ l.b.

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First Some Definitions

- The depth of a node is its distance to the root
- An Up edge is an edge from a node of larger depth to a node of smaller depth
- A Level edge is an edge between two nodes of same depth
- A Down edge is an edge from a node at smaller depth to a node at higher depth
- A leveled graph of a social network is the graph of Up edges

Excitation Paths

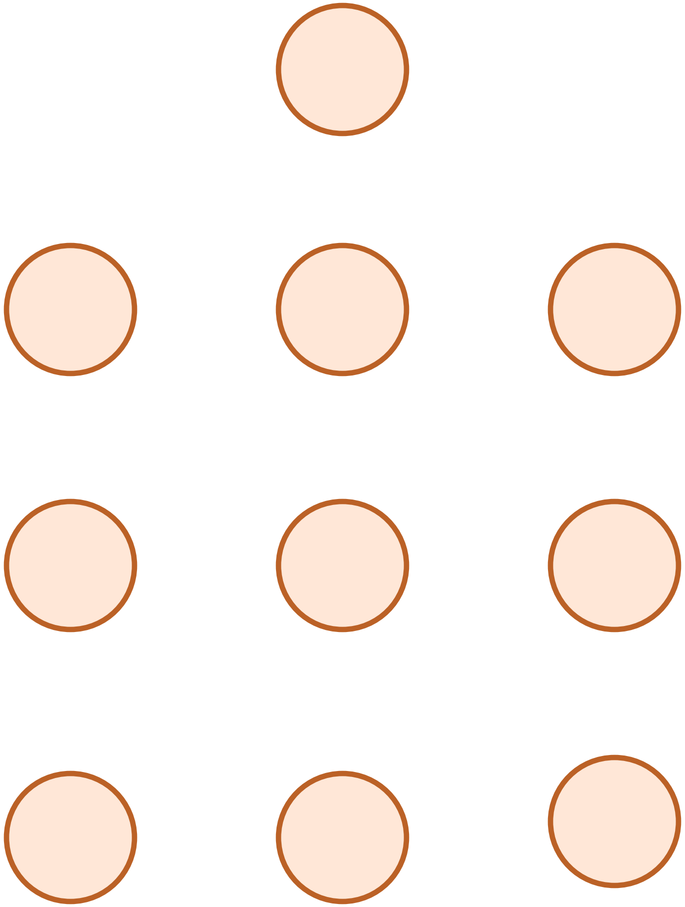
- An excitation path for a node n is a VIQ in which a subset of the free agents form a simple directed path from n to the output. All agents not on the path with inputs into the path are suppressed.

- We also have a shortest excitation path

The Learning Algorithm For Networks Without 1 Edges

- First Find-Up-Edges to learn the leveled graph of S
- For each level, Find-Level-Edges
- For each level, bottom-down, Find-Down-Edges

FIND-UP-EDGES



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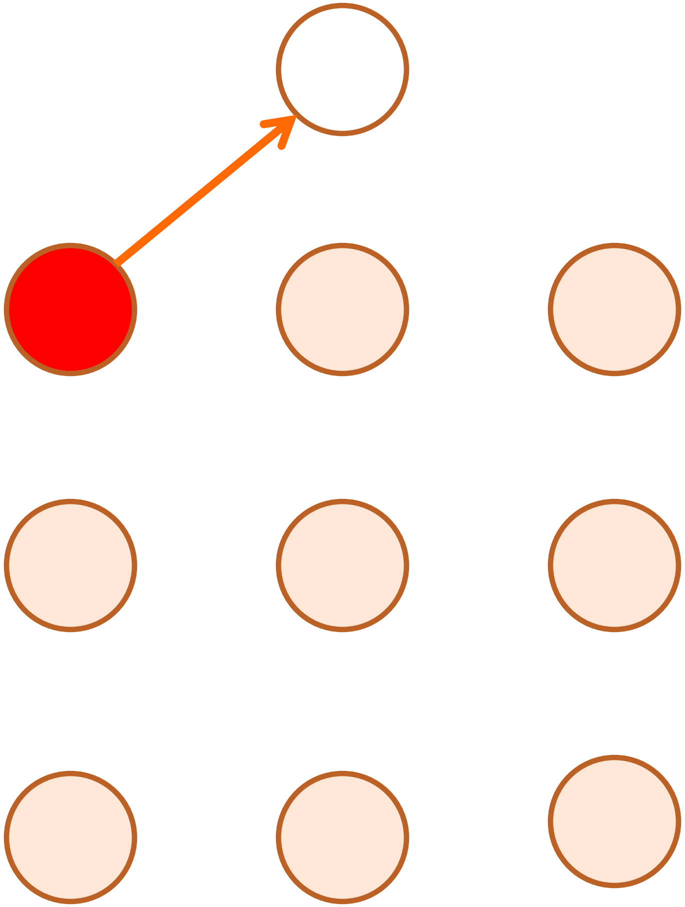
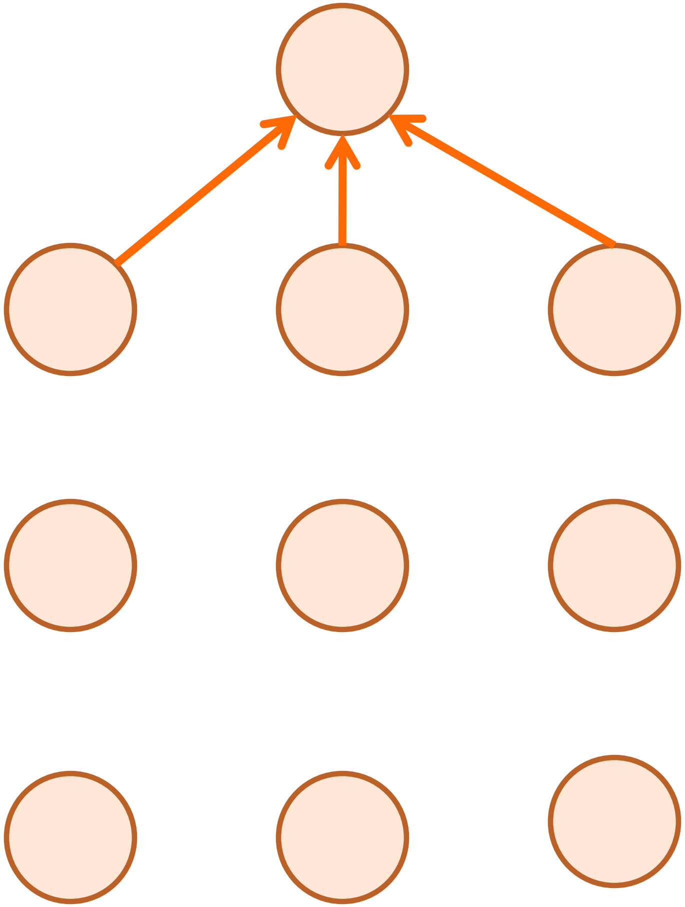


Find-Up-Edges



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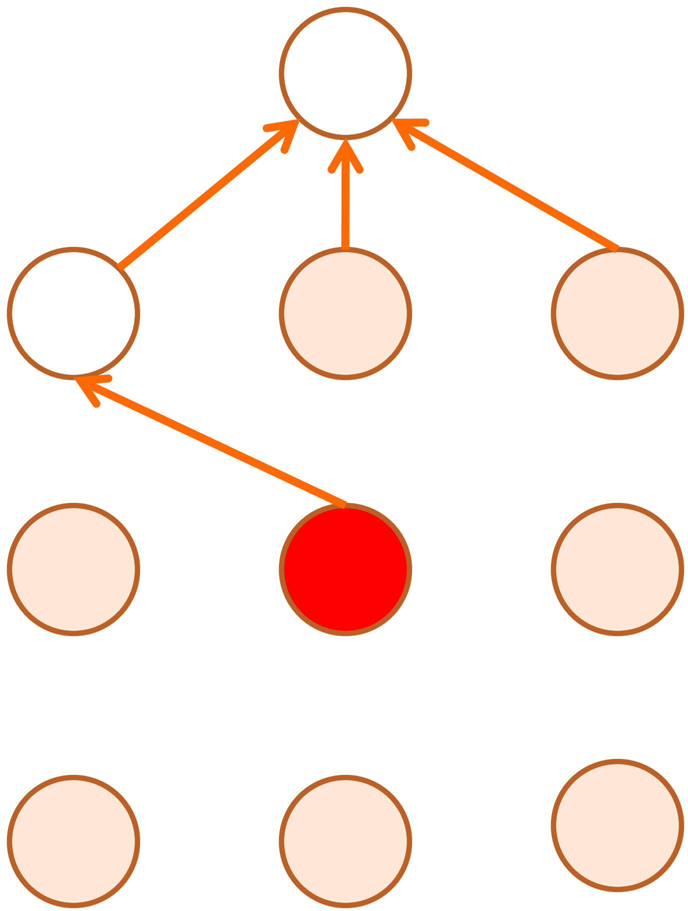

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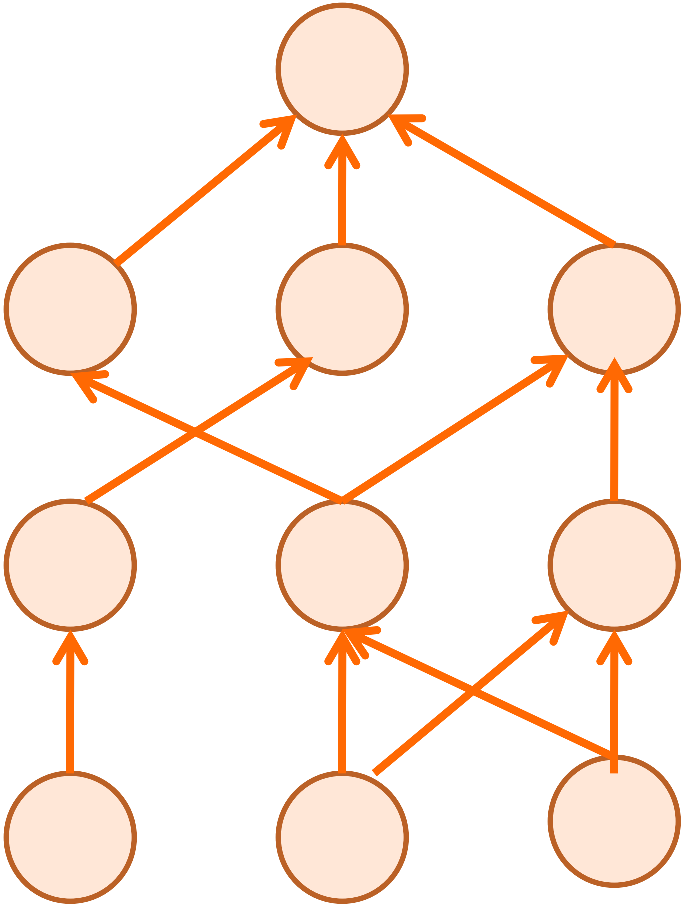
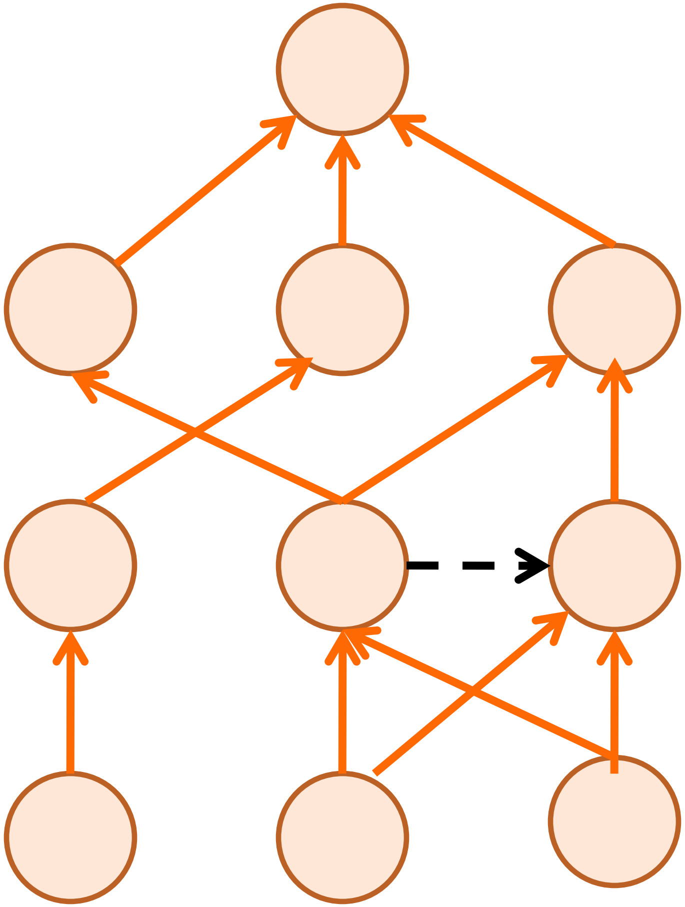


Find-Level-Edges



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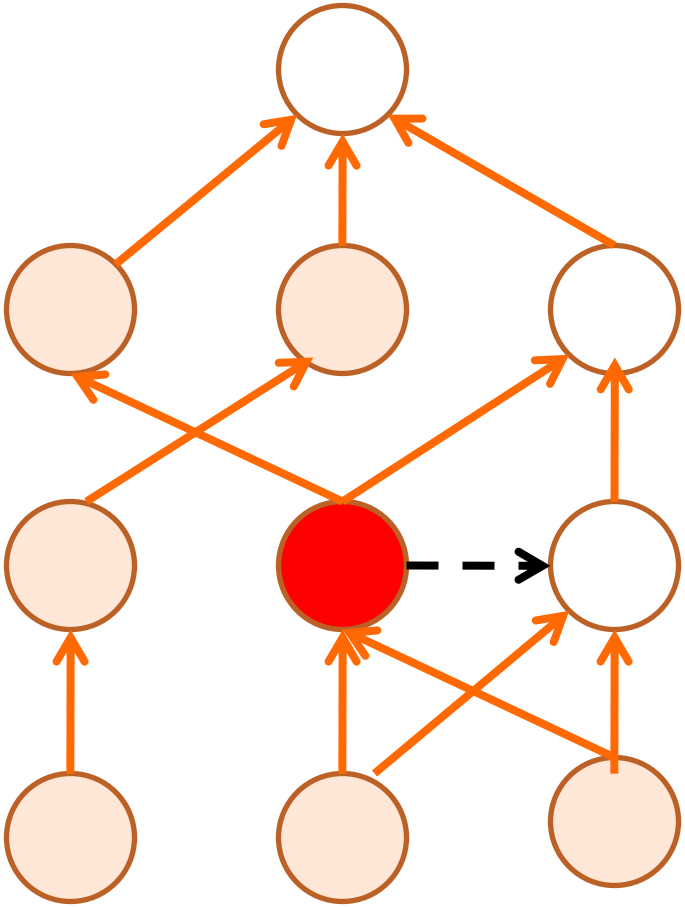

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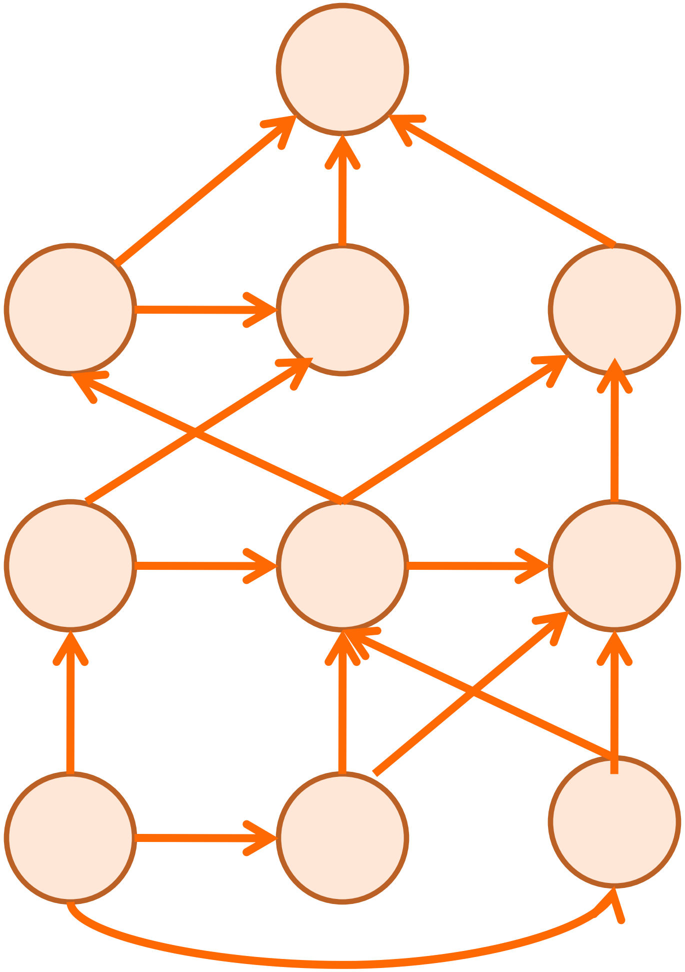
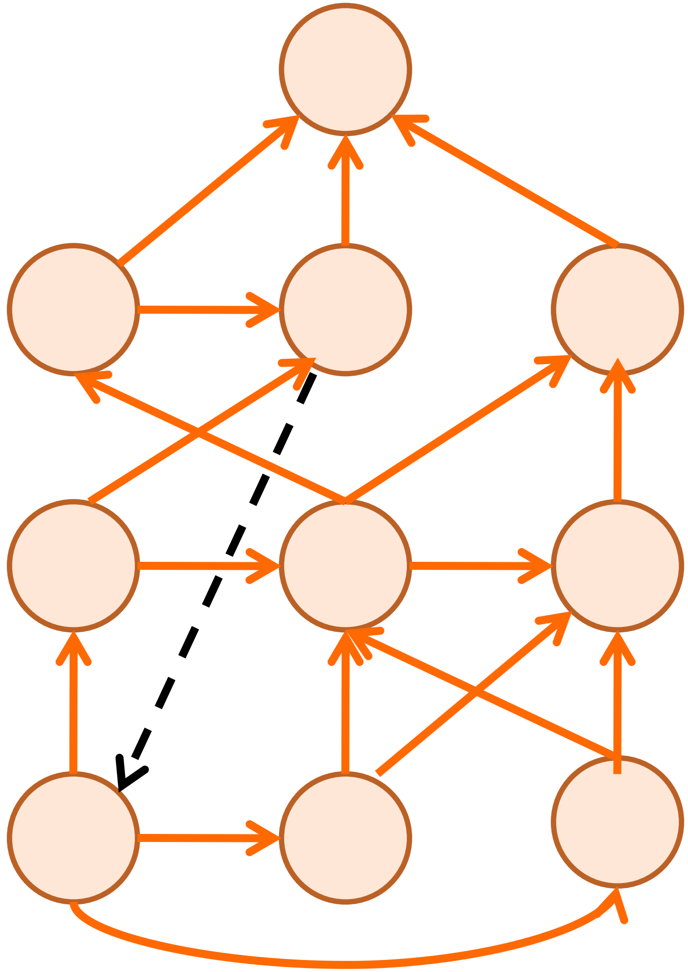


Find-Down-Edges



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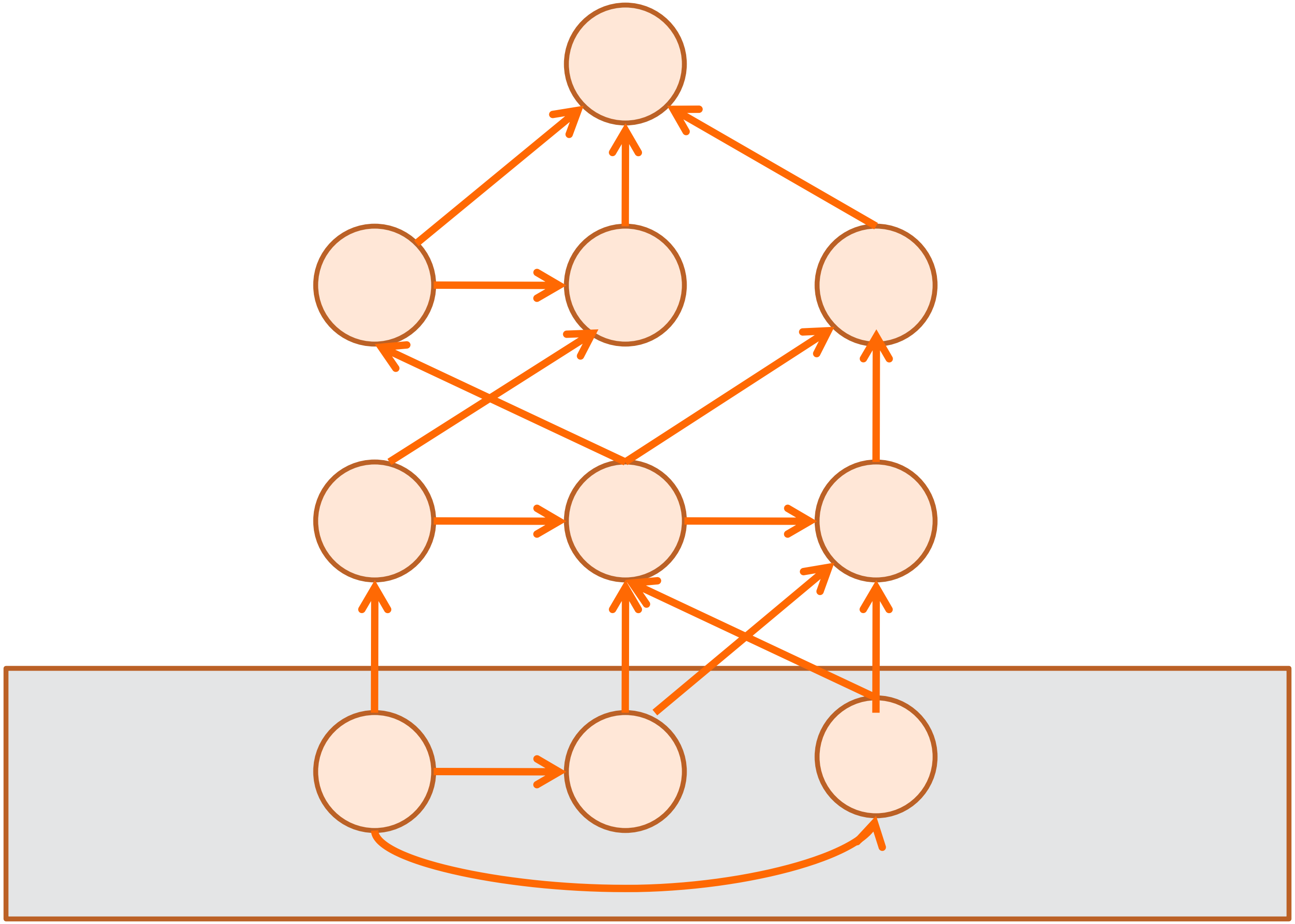

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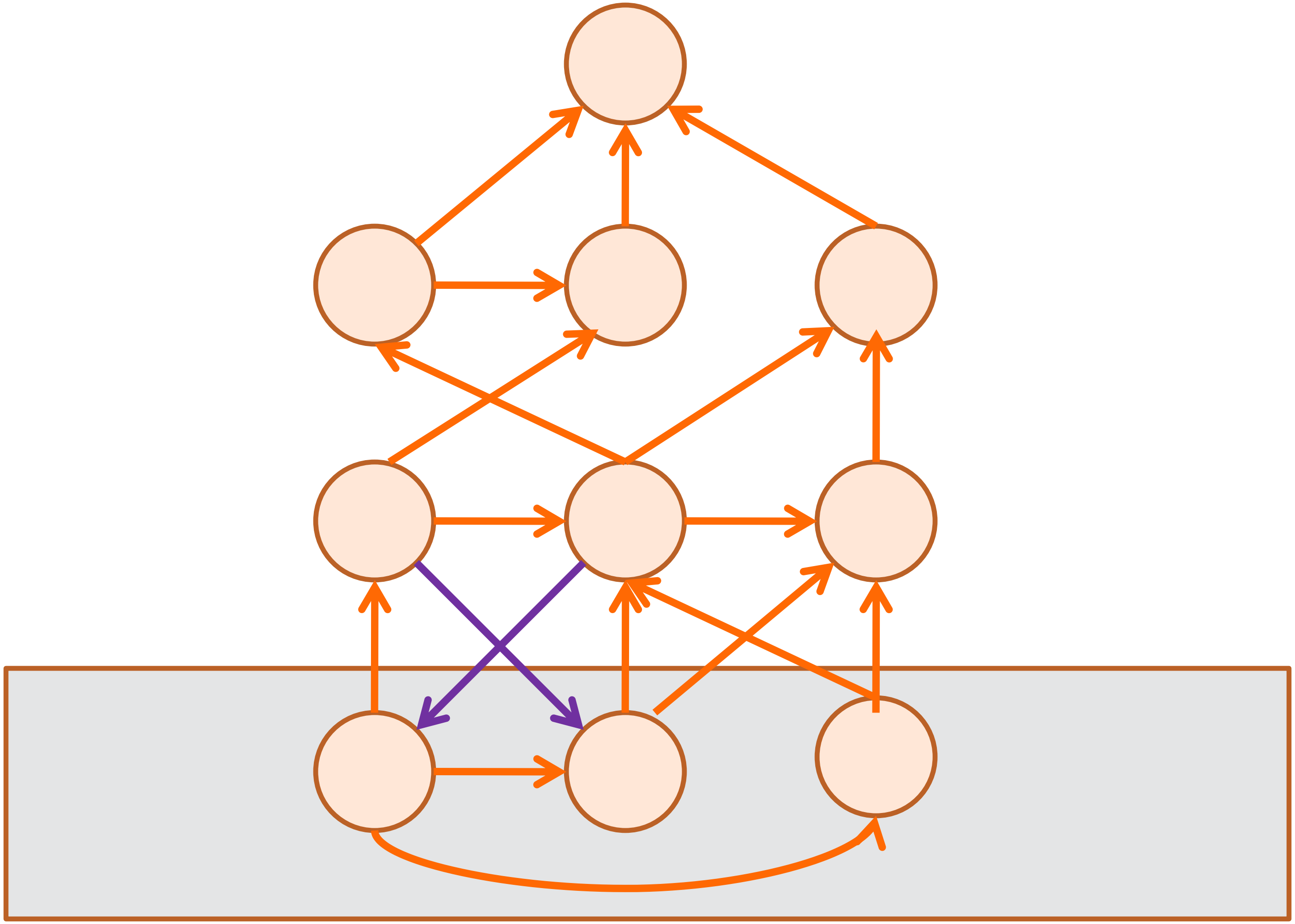

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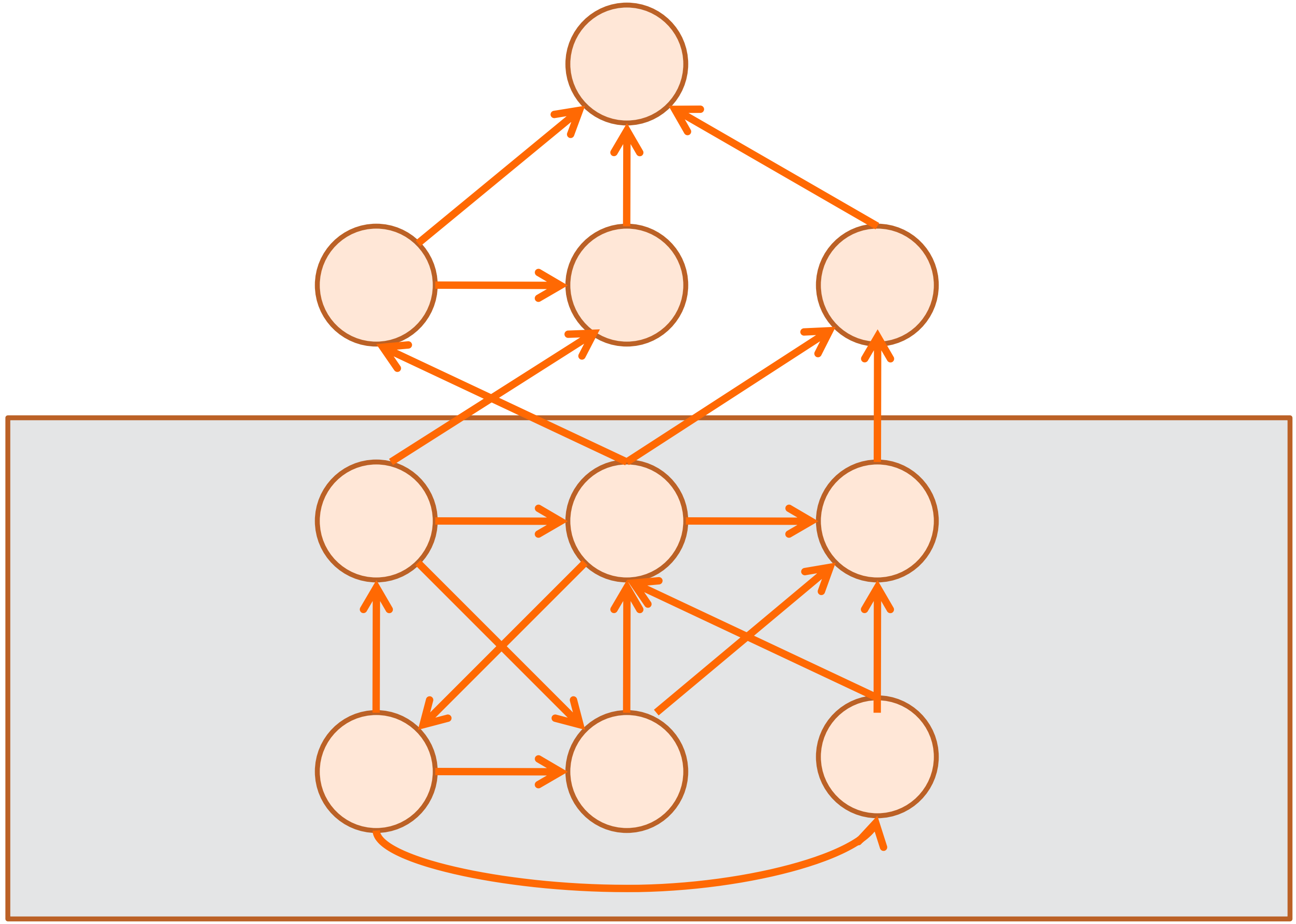

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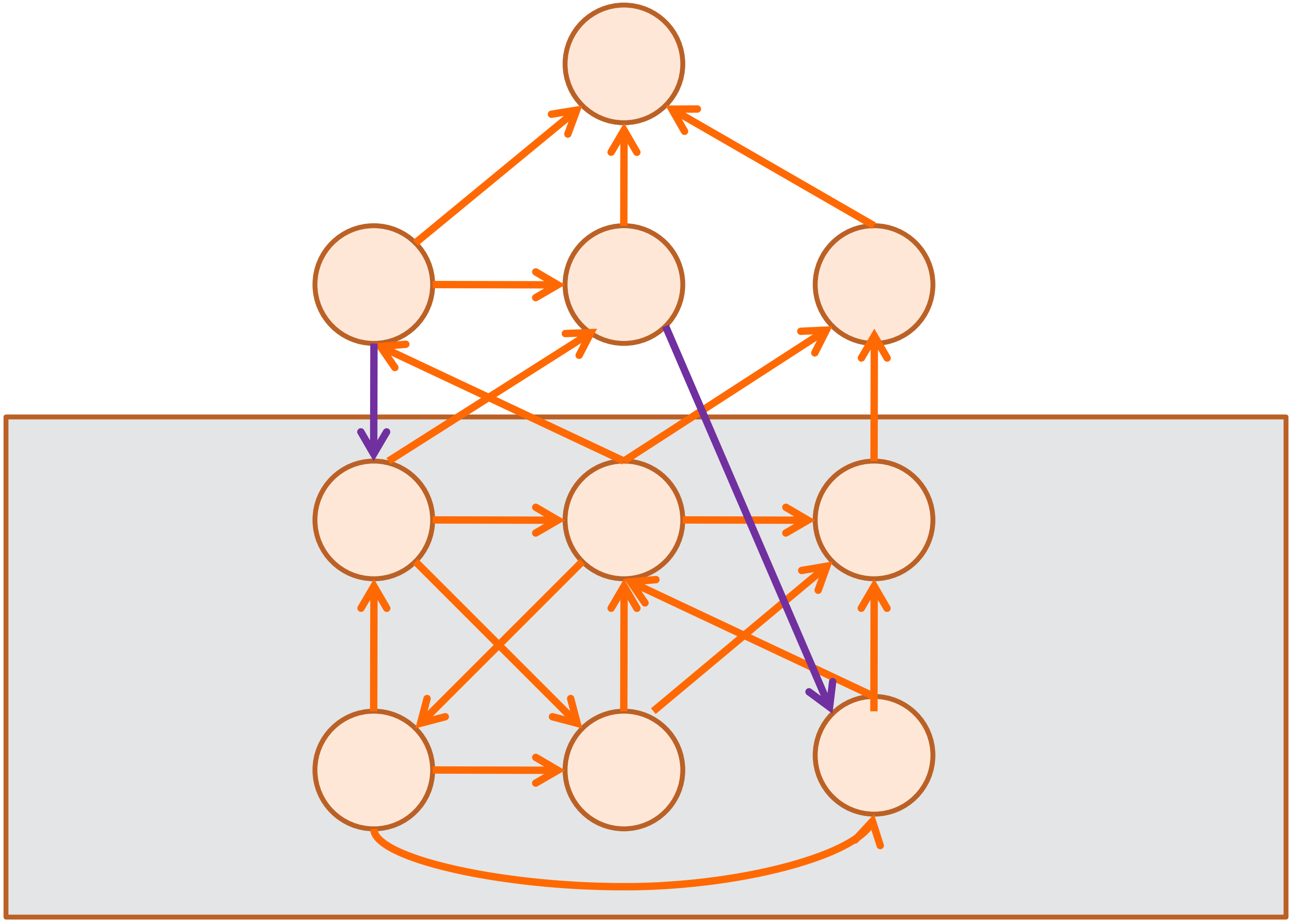

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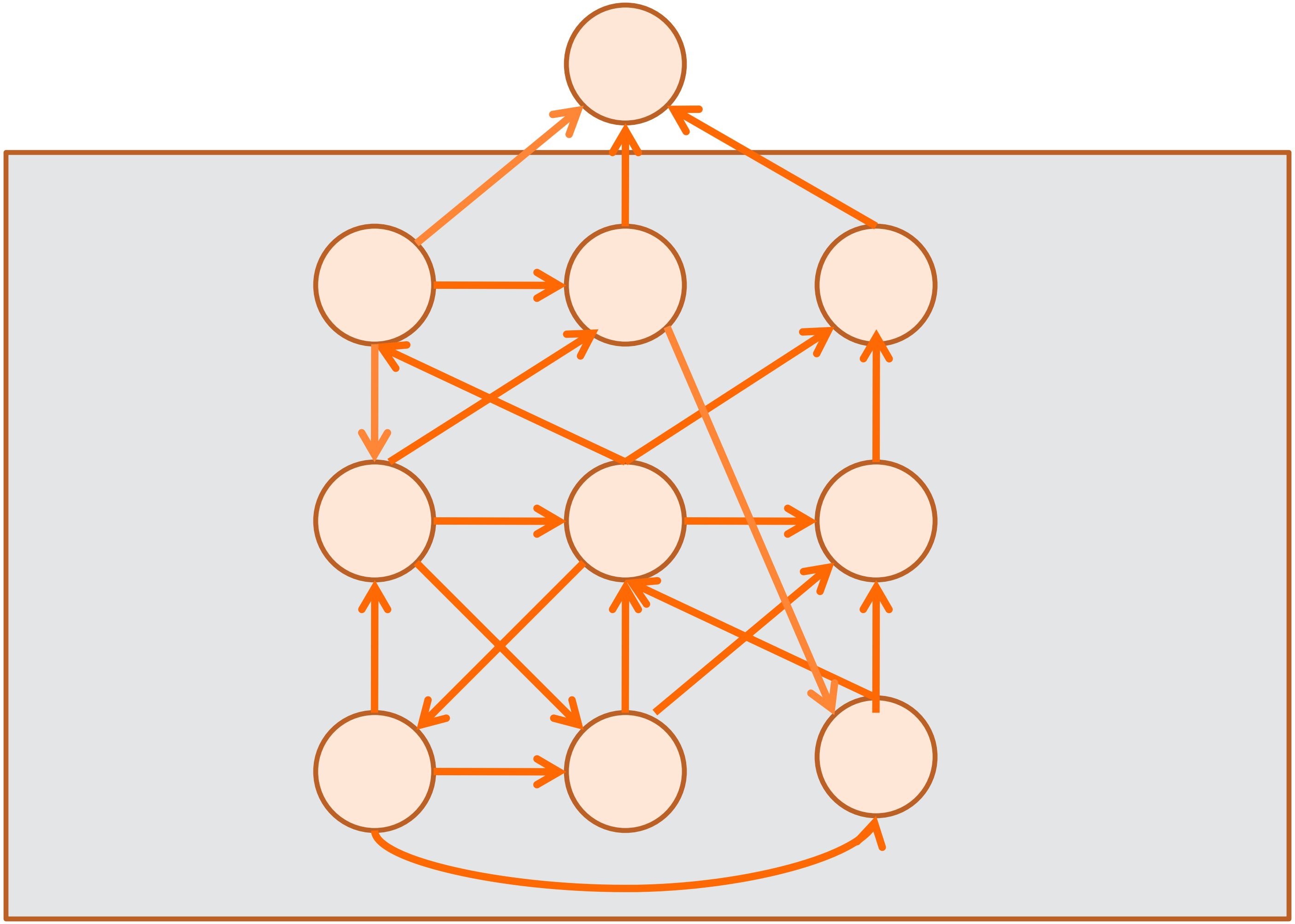

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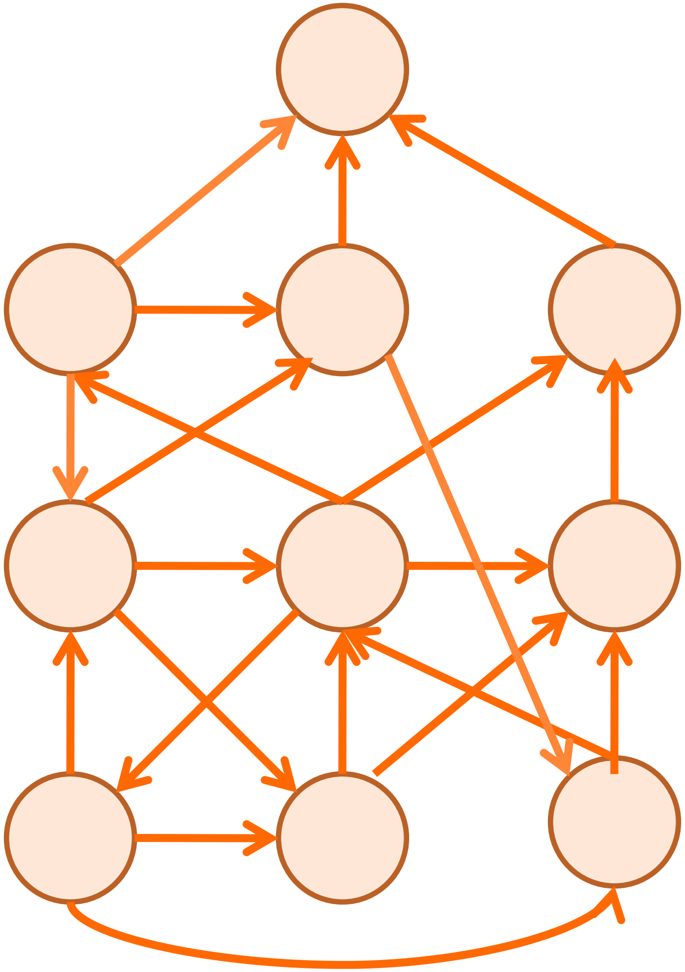
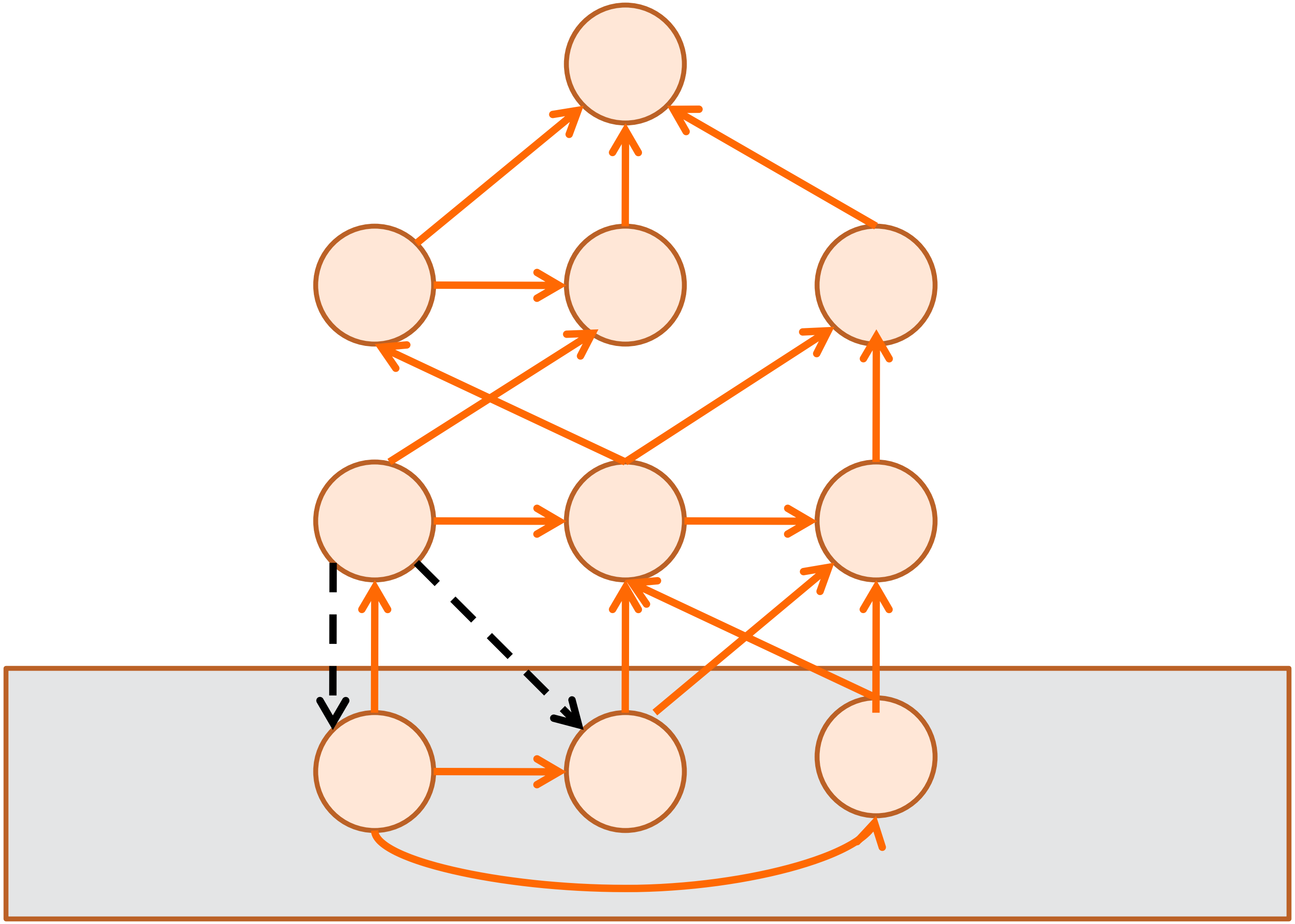


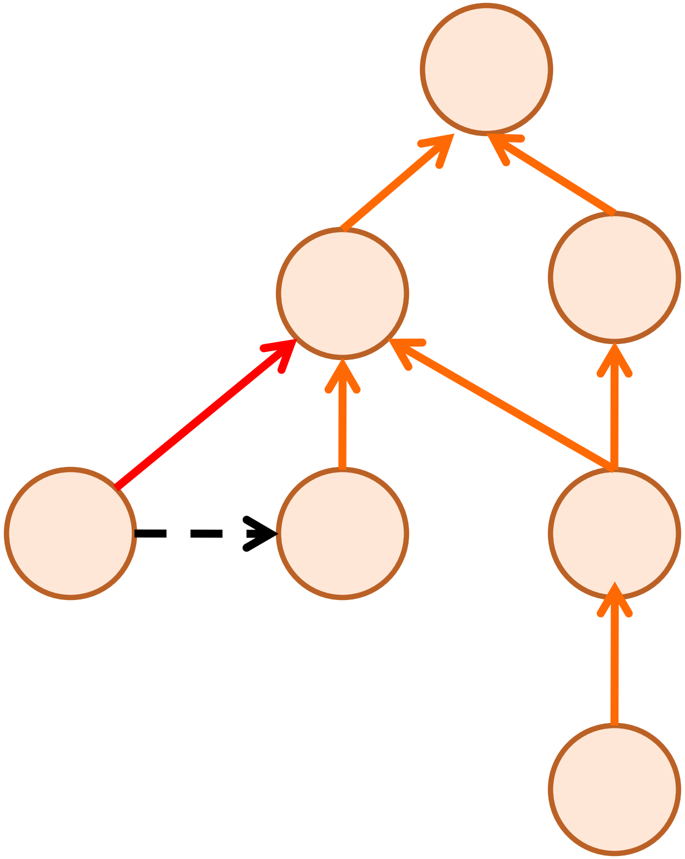
Find-Down-Edges

- For each node u at current level
  - Sort each node v_i in C (complete set) by distance to the root in $G – \{u\}$
  - Let $v_1 \dots v_k$ be the sorted v_is
  - Let $pi_1 \dots pi_k$ be their corresponding shortest paths to the root in $G – \{u\}$
  - For i from 1 to k
    - Do experiment of firing u, leaving pi_i free, and suppressing the rest of the nodes.

For Example



With Ones – a Problem



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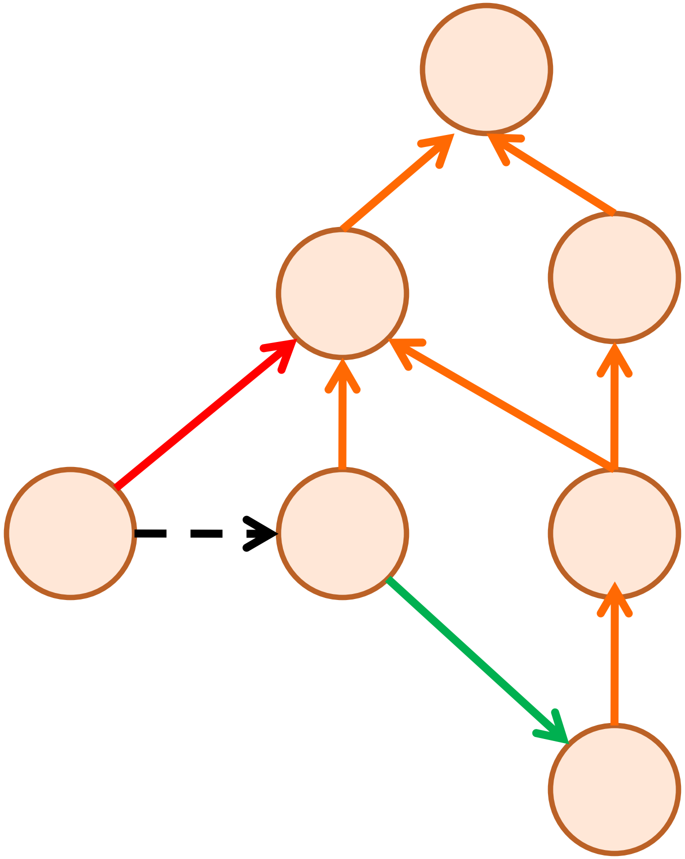

With Ones

- Algorithm gets more complicated
- Level edges and down edges are found in one subroutine
- In looking for down edges from u, need to avoid not just u, but also all nodes reachable from u by 1 edges
- There always exists some pair of nodes, with source in L (current level) and destination in C + L where you can look for an edge.

In the End

- We do 1 query per each possible edge, giving an $O(n^2)$ algorithm
- Matches the $\Omega(n^2)$ lower bound

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Finding Influential Nodes

- Suppose instead of learning the social network, we wanted to find an influential set of nodes quickly.

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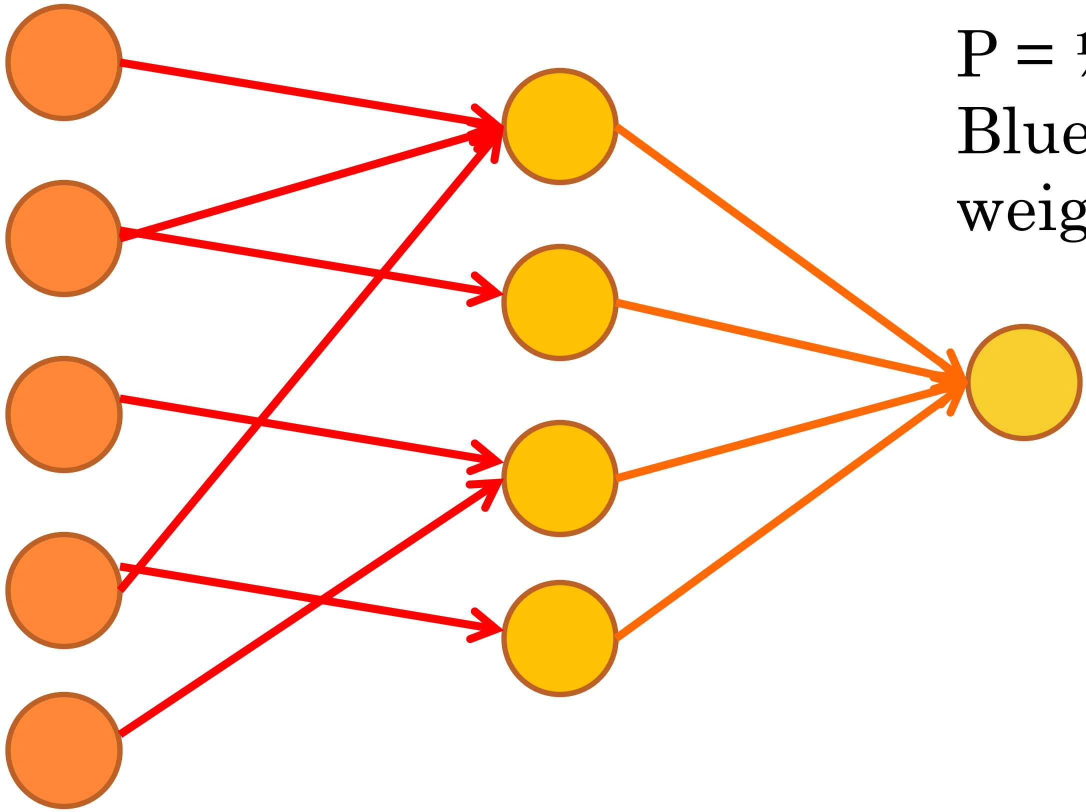
Finding Influential Nodes

- Suppose instead of learning the social network, we wanted to find an influential set of nodes quickly.
- A set of nodes is influential if, when activated, activates the output with probability at least p
- NP Hard to Approximate to log n, even if we know the structure of the network

Reduction from Set Cover



P = ½
Blue edges have weight (1- ½ ^(1/k))

sets

k elements

An Approximation Algorithm

- Say the optimal solution has m nodes
- Suppose we wanted to fire the output with probability (p – ε)
- Let I be the set of chosen influential nodes.
- Observation: at any point in the algorithm, greedily adding one more node w to I makes

$$S(e_{I \cup \{w\}}) \geq S(e_I) + \frac{p - S(e_I)}{m}$$

Analyzing Greedy

- Using a greedy algorithm, we let k be the number of rounds the algorithm is run

For

$$p\left(1-\frac{1}{m}\right)^k < \epsilon$$

it suffices that

$$e^{-\frac{k}{m}} < \frac{\epsilon}{p}$$

or

$$k > m\log\left(\frac{p}{\epsilon}\right).$$

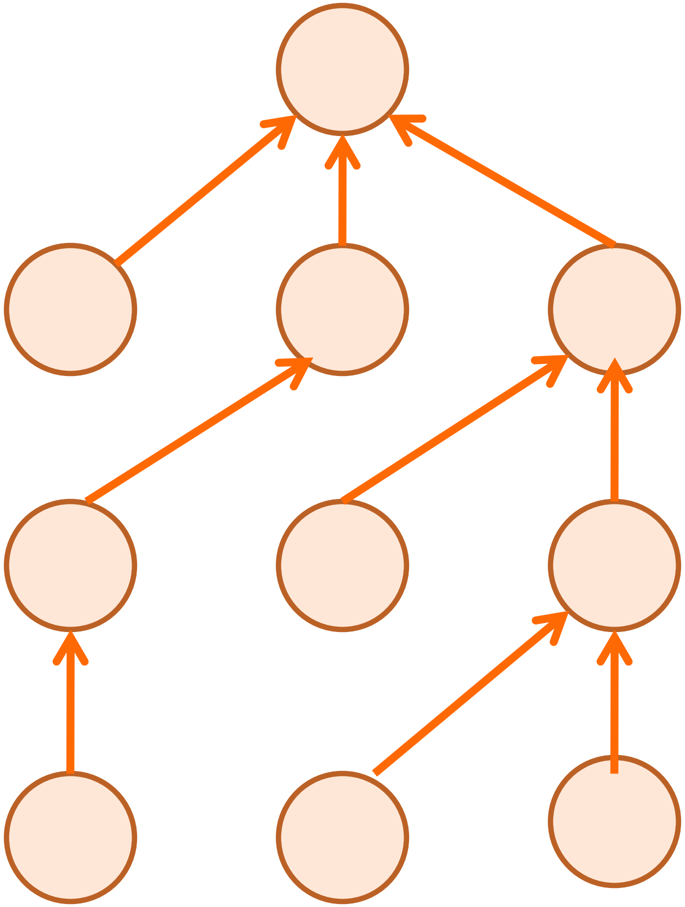
Reconciling Greedy

- Therefore after m $\log(p/\varepsilon)$ rounds, we get to within ε of p.

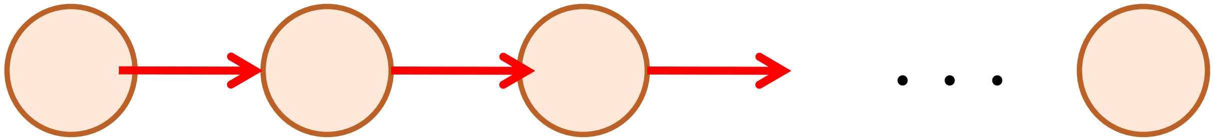
- To reconcile with the set cover reduction, if we set $\varepsilon = ½^{(1/n)} - ½^{(1/(n-1))} = \theta(1/n^2)$, this forces us to cover all the elements.

- Giving a m $\log (p\ n^2) = O(m \log (n))$ approximation. Matches the set cover lower bound.

Trees



Lower Bound



Like sorting with comparisons

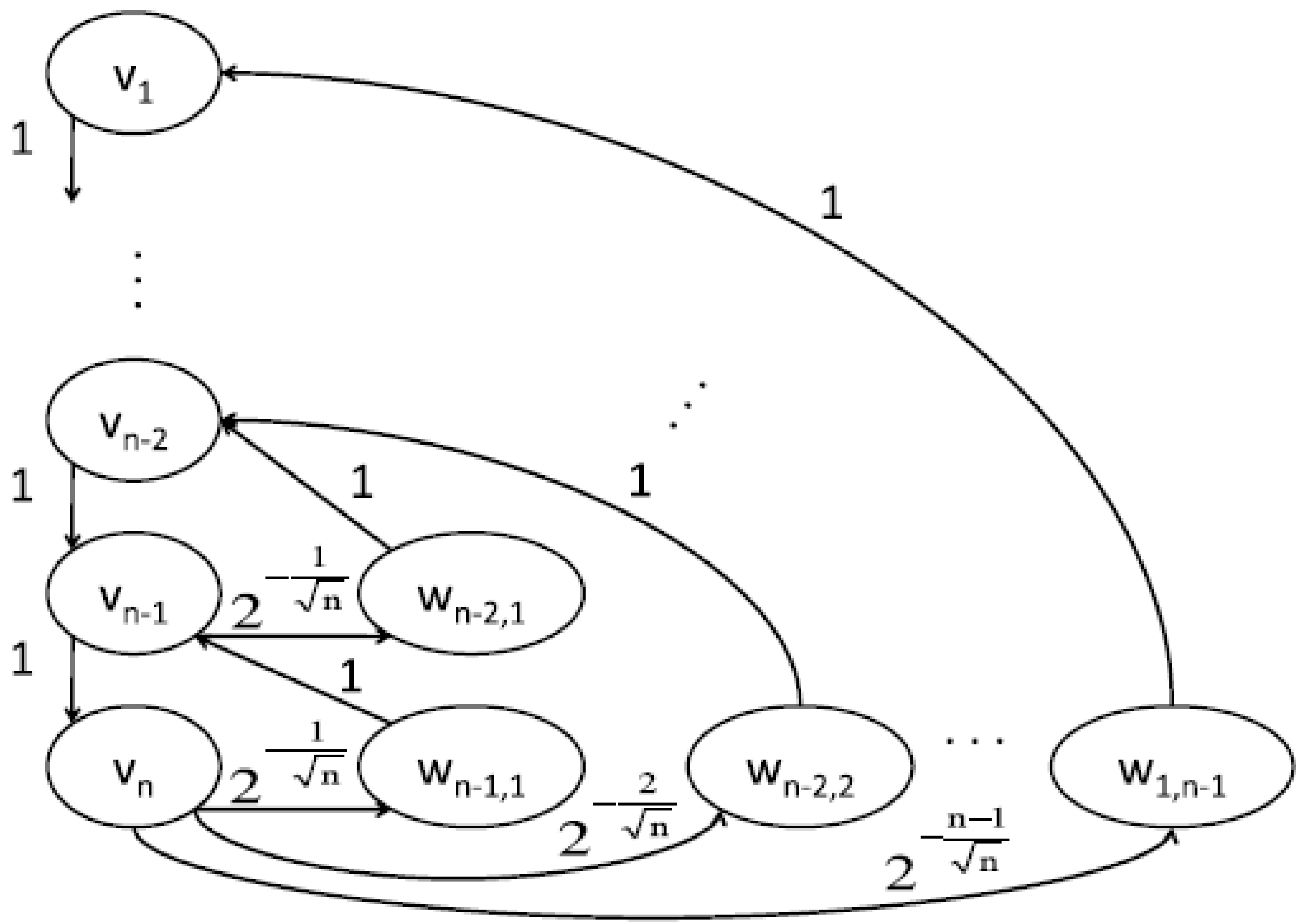
Gives a Ω(n log n) lower bound

The Idea for Algorithm

- Ancestor Test. To test if u has an ancestor in set S, fire u and suppress all nodes in S.

- We can build up a tree by adding one node at a time and binary searching where it belongs in the tree using ancestor tests.

If We Don't Have Exact VIQs



Discussions and Open Problems

- Interesting model to study various hidden structures.
- Finding non-path based methods for social networks. (Actually also an open question for learning circuits with VIQs)
- Reducing Query Size
- Other models.