Learning Acyclic Probabilistic Circuits Using Test Paths

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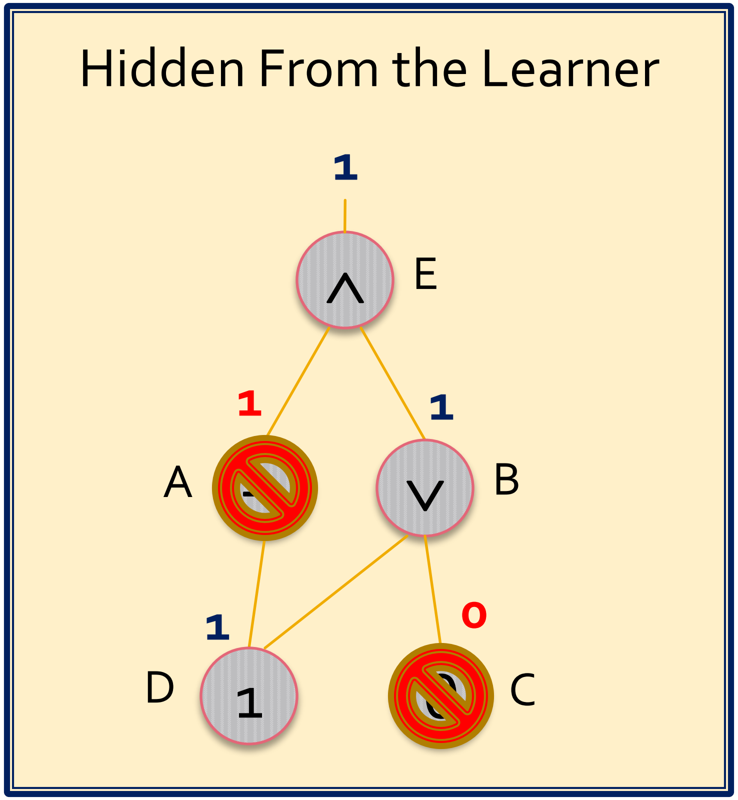
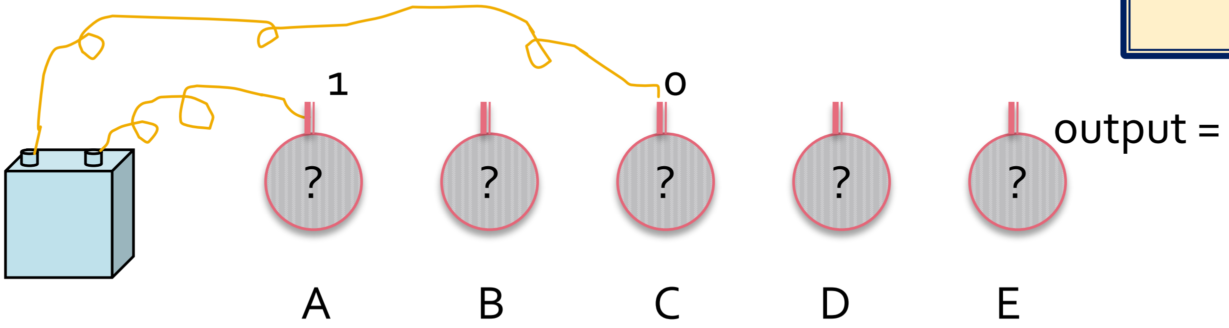
COLT 2008

Value Injection Queries (VIQs)

- Introduced by [AACW ’06]
- Experiments on a hidden Circuit.
  - a gate output may be fixed
  - a gate may be left free
- Query
  - given an experiment, we can observe its output
- Example:





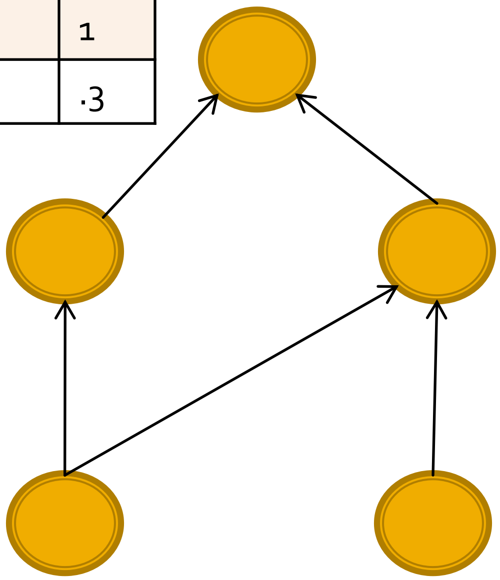
Motivation for the Model

- Model proposed in AACW '06 to study gene disruption and overexpression in gene interaction networks – Boolean circuits.

- Genes have more than two states: AACR '07 studied large alphabet circuits, interested in sensitivity on various parameters

- Real life circuits are often probabilistic – brings us to this paper AACER '08

(Acyclic) Probabilistic Circuits



Input 1	Input 2	P(1)
0	0	.15
0	1	.5
1	0	1
1	1	.3

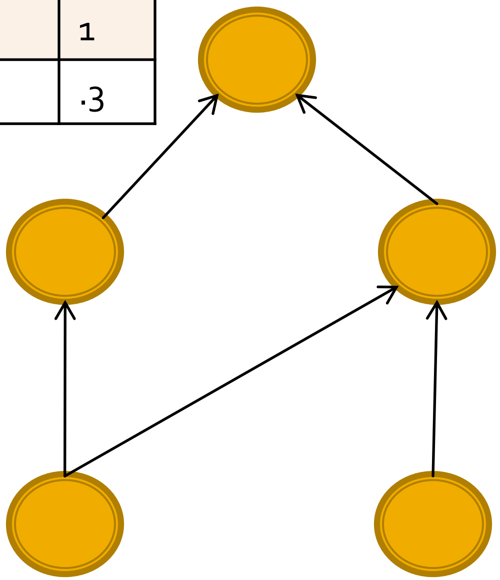
Input 1	Input 2	P(1)
0	0	.7
0	1	1
1	0	.5
1	1	0

Input	P(1)
0	.4
1	.8

P(1)
.9

P(1)
.9

(Acyclic) Probabilistic Circuits



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VIQs on Probabilistic Circuits

Exact VIQs

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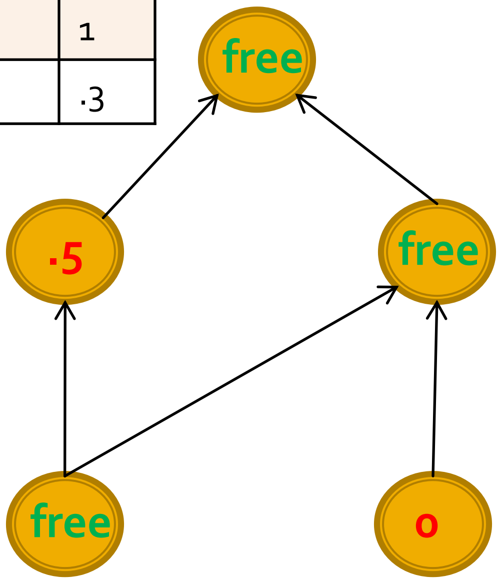

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~~0~~	~~4~~
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~~P(1)~~
~~9~~

VIQs on Probabilistic Circuits

Exact VIQs

The Learning Problems

- **ε-Approximate Learning**
  - **ε-behavioral equivalence:** Circuits C and C’ are ε-behaviorally equivalent if for any experiment s, $d(C(s)-C'(s)) < \varepsilon$.
    - $d(C(s)-C'(s))$ is a notion of statistical distance
  - **The problem:** Given query access to a hidden circuit C^*, find a circuit C ε-behaviorally equivalent to C^* by making value-injection queries.

- **Exact Learning**
  - **behavioral equivalence:** Two circuits C and C’ are behaviorally equivalent if for any experiment s, $C(s)=C'(s)$.
  - **The problem:** Given query access to a hidden circuit C^*, find a circuit C behaviorally equivalent to C^* by making exact value-injection queries.

Previous Work

Circuit	Fan-in	Topology	Gates	VIQ Learnability
Boolean	2	arbitrary	AND/OR	NP-Hard
Boolean	unbounded	constant depth	AND/OR/Θ_2	NP-Hard
Boolean	constant	log depth	arbitrary	Poly-time
Large ∑	constant	log depth	arbitrary	W(1) Hard in shortcut width
Large ∑	constant	bounded shortcut width	arbitrary	Poly-time
Analog	constant	bounded shortcut width	arbitrary	Poly-time approximate

Talk Outline / Main Results

- The Test Path Lemma

- Boolean Probabilistic Circuits
  - Approximately Learnable

- Larger Alphabet Probabilistic Circuits
  - Not Learnable Using Test Paths
  - Learnable with Function Injection Queries

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If nothing else, I want to show you how probabilistic circuits behave differently than you might expect

The Test Path Lemma

- A test path for a wire w is a value injection experiment in which the free gates form a directed path in the circuit graph from w to the output wire. All the other wires in the circuit are fixed, including the inputs of w.

- The test path lemma: Let C be a deterministic circuit.

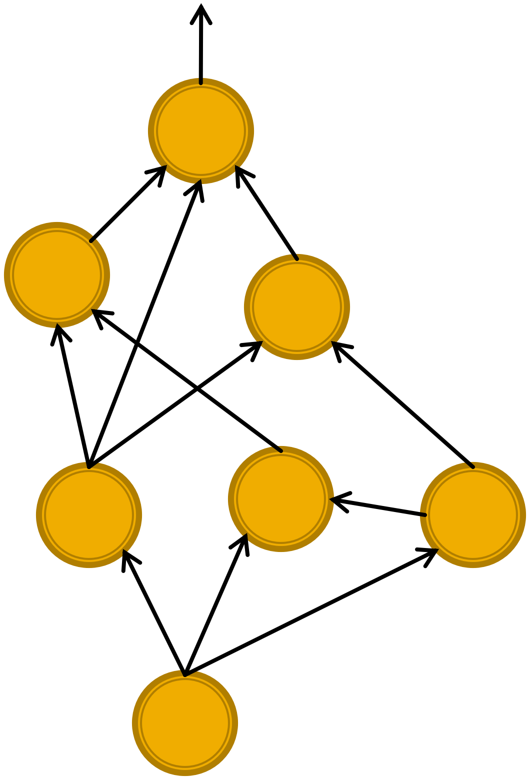
  If for some value injection experiment e, wire w and alphabet symbols σ and τ it is the case that

  $C(p|_{w=\sigma}) = C(p|_{w=\tau})$

  Then for every test path p < e, then also

  $C(e|_{w=\sigma}) = C(e|_{w=\tau})$.

Test Path Lemma Illustrated



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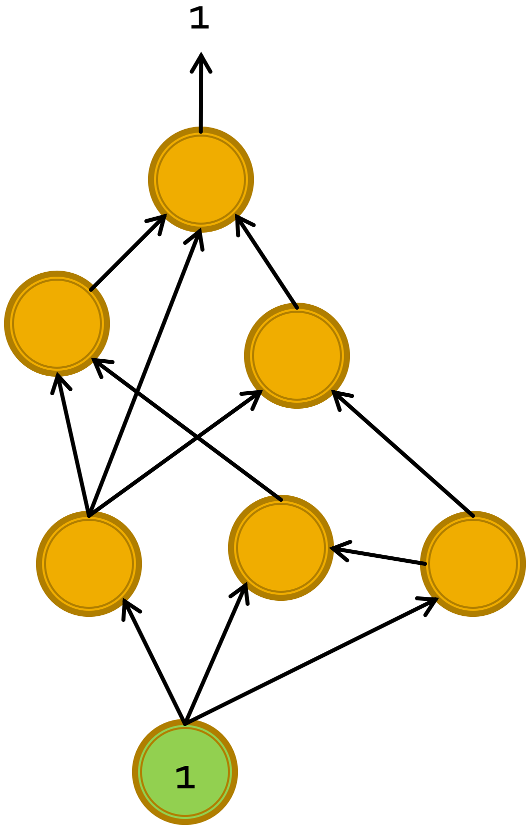

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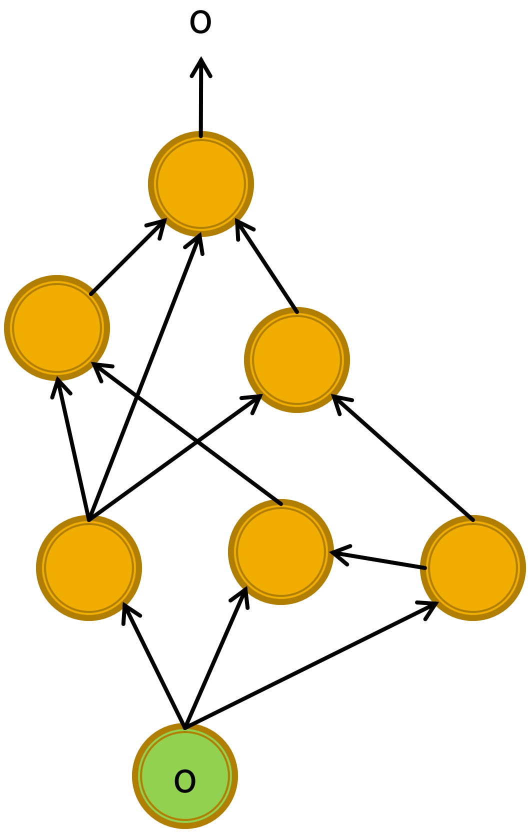

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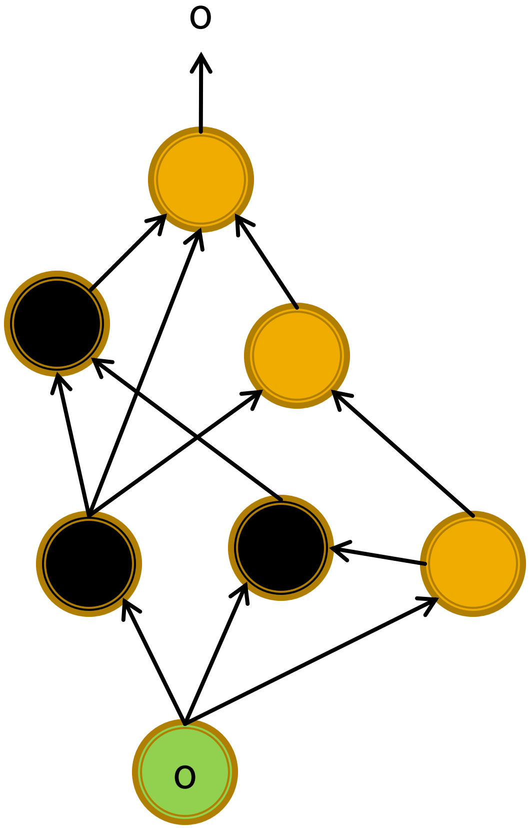

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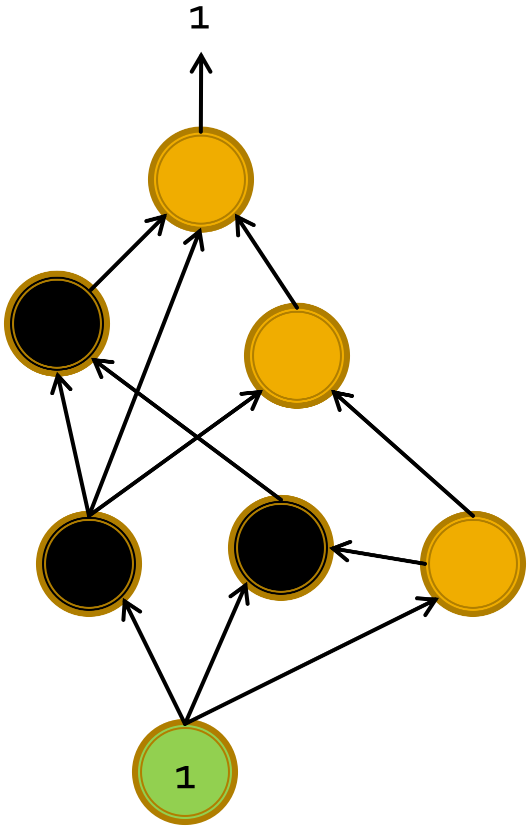

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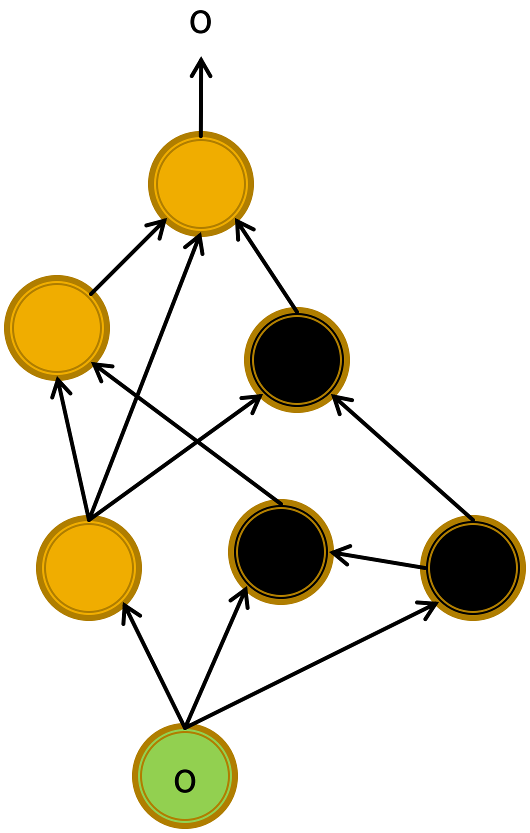

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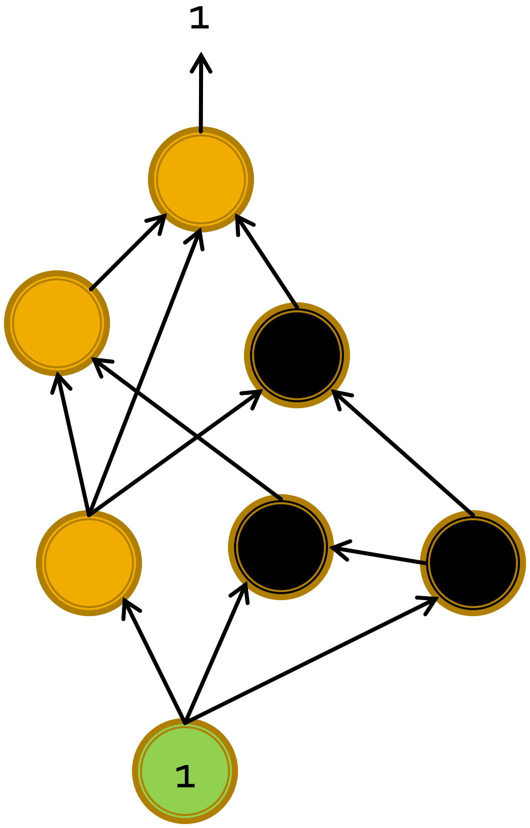

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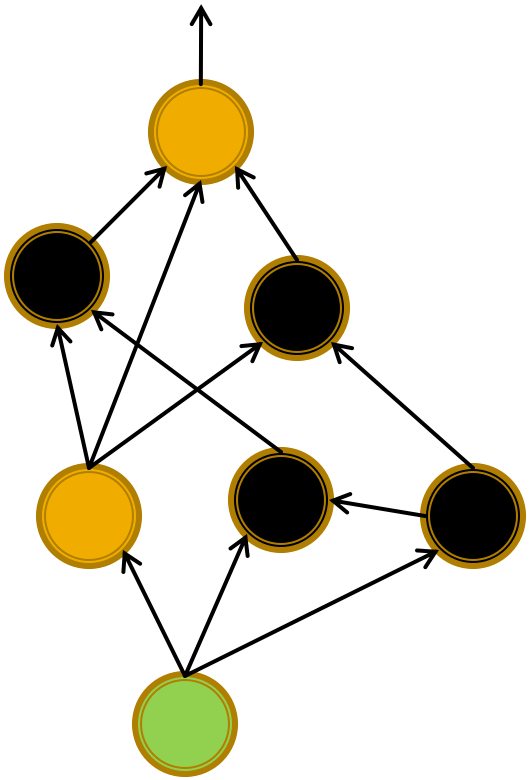
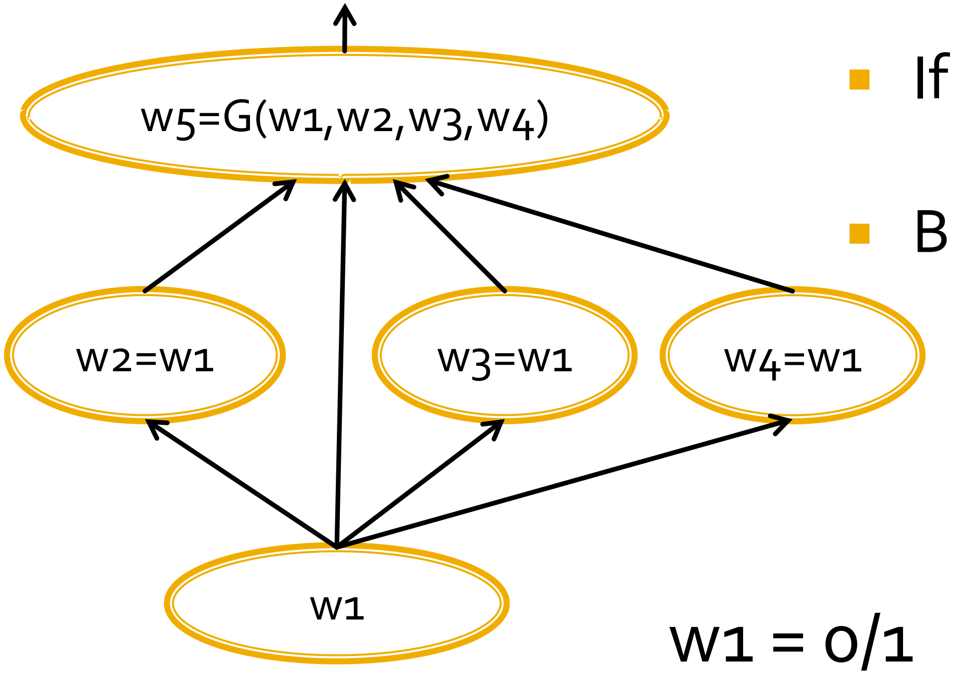


Attenuation of Signal in Test Paths

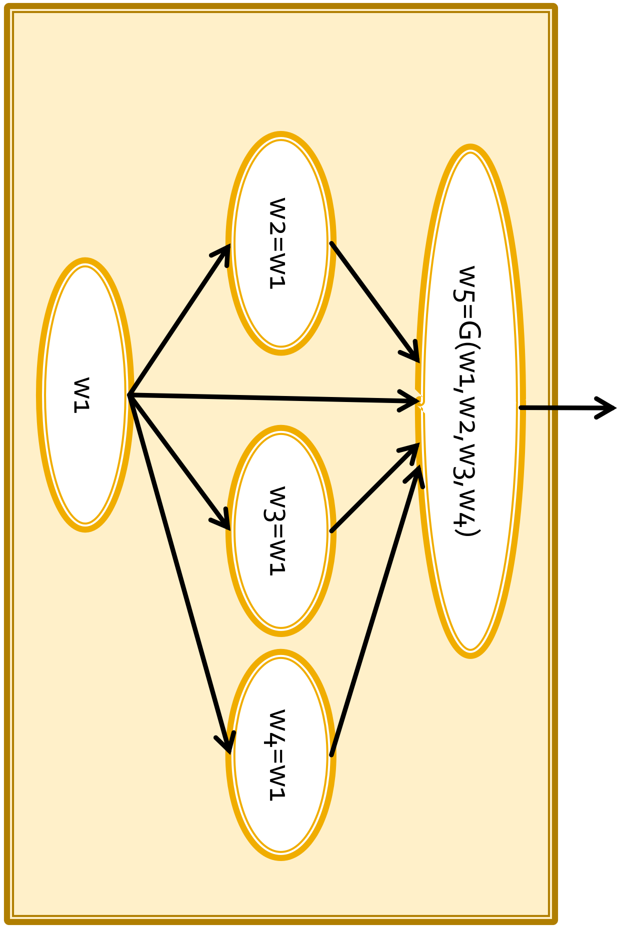
Let $G(w1,w2,w3,w4) = ((1-w1)+2w2+2w3+2w4)/7$



$w1 = 0/1$

- If e sets all wires to be free, then $d(D1(e|_{w=0}),D1(e|_{w=1})) = 5/7$.
- But for any test path p for w1 $d(D1(p|_{w=0}),D1(p|_{w=1})) = 1/7$.

Exponential Attenuation



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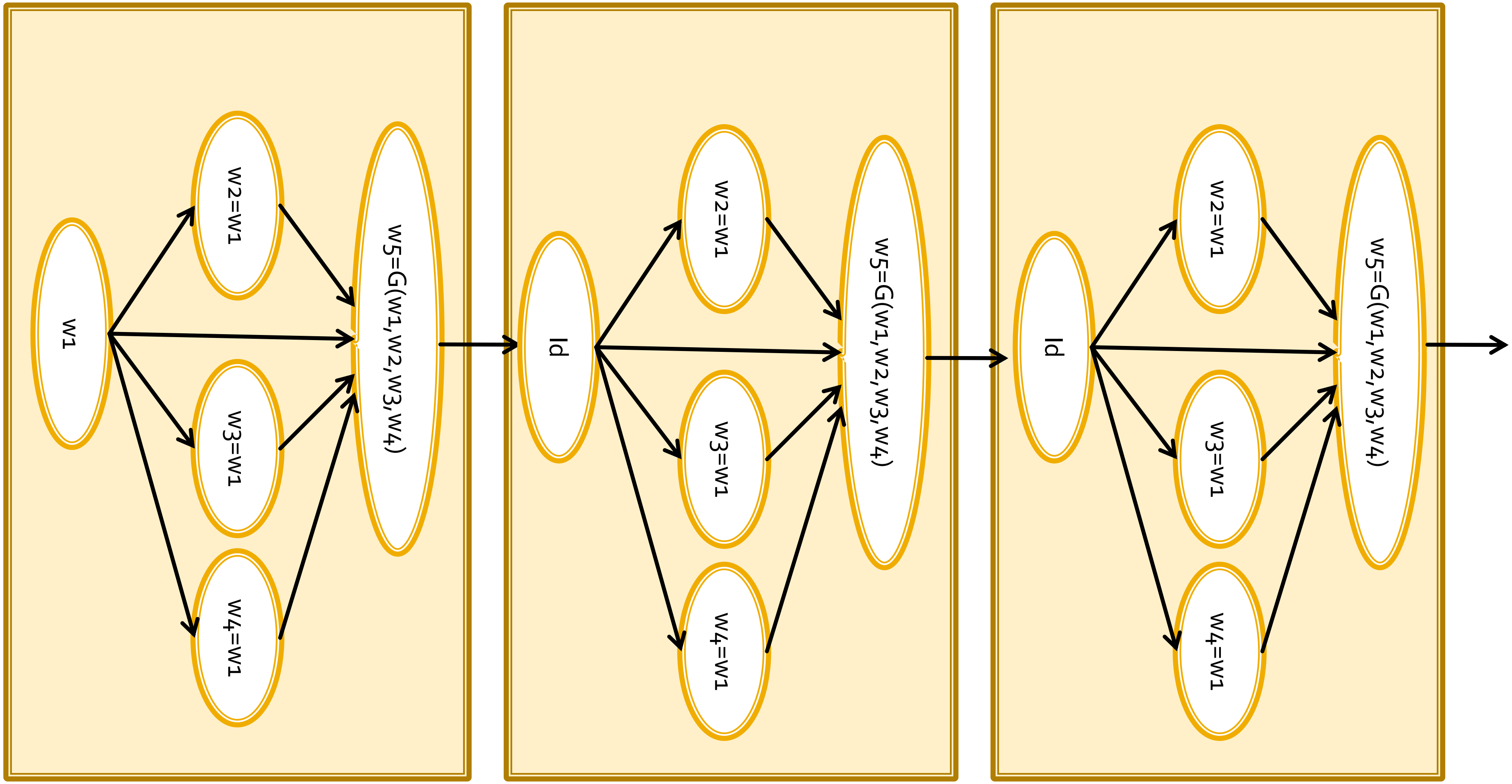


Boolean Probabilistic Circuits

- Positive Result for Boolean Probabilistic Circuits:
  - There is a nonadaptive learning algorithm that with probability at least (1 – δ) ε-approximately learns any Boolean probabilistic circuit w/ n wires, constant fan-in and depth c log n using value injection queries in time bounded by a **polynomial in n, 1/ ε and log(1/ δ).**

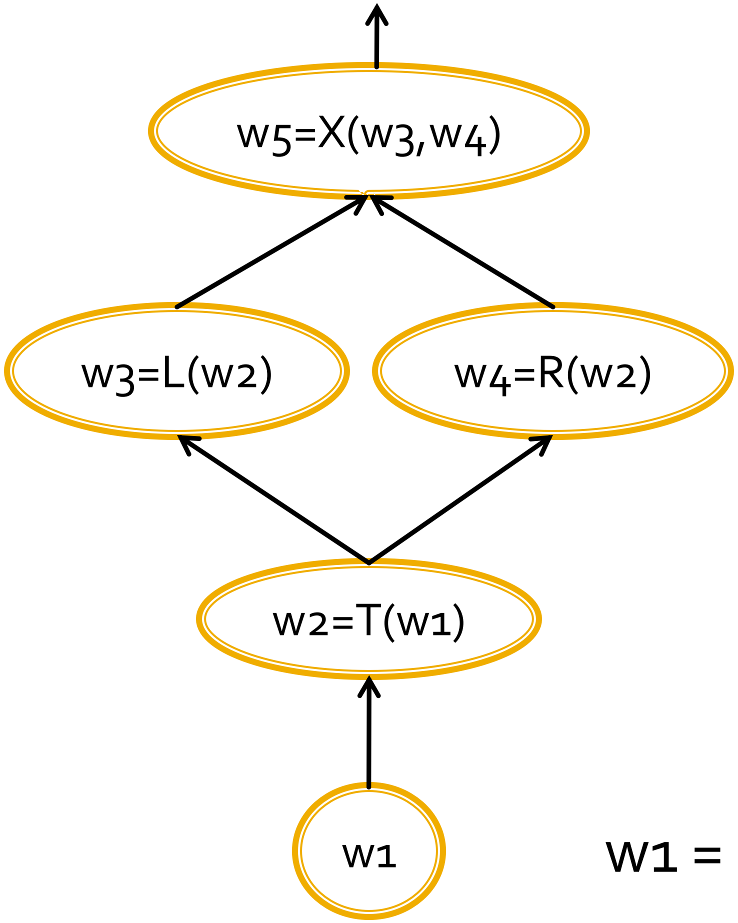
Boolean Probabilistic Circuits

- Positive Result for Boolean Probabilistic Circuits:
  - There is a nonadaptive learning algorithm that with probability at least (1 – δ) ε-approximately learns any Boolean probabilistic circuit w/ n wires, constant fan-in and log n depth using value injection queries in time bounded by a **polynomial in n, 1/ ε and log(1/ δ).**
- Proof idea:
  - The test paths lemma still holds (with some attenuation)
  - The test paths approximately determine the function of the circuit. Collect all possible test paths (**of log depth**) and put them into CircuitBuilder (AACW '06)

Larger Alphabet Probabilistic Circuits

- Lets consider probabilistic circuits that have gates that operate on more than two alphabet symbols.
- What happens to the test path lemma in the large alphabet, probabilistic case?

Test Paths Fail (Completely) for |∑|>2



w5=X(w3,w4)

w3=L(w2)

w4=R(w2)

w2=T(w1)

w1

T(00)=T(11) =	U({00,11})
T(01)=T(10) =	U({01,10})
L(00)=L(01)=	00
L(10)=L(11)=	01
R(00)=R(10)=	00
R(01)=R(11)=	01
X(ab,cd)=	0(b⊗d)

w1 = 00 / 01

Function Injection Queries

- An alphabet transformation is a function f that maps symbols to distribution over symbols.
- A function injection experiment is a mapping that for each wire either leaves it free, assigns it an alphabet symbol, or assigns a transformation f.
- A function injection query (FIQ) takes a function injection experiment and returns the symbol assigned to the output wire.

Learning Large Alphabet Circuits

- A 2-partition experiment is a function injection experiment in which every alphabet transformation is a 2-partition.

- By using 2-partition experiments, we can “smash” the large alphabet circuits back to the Boolean case.
  - We get same positive learnability results for probabilistic large alphabet circuits using FIQs as we have for probabilistic Boolean circuits using VIQs.

Open Problems

- Can we learn large alphabet probabilistic circuits without using test paths? This seems very hard to approach…
- Can “NOT injections” help?
- We can study other variants of this model, for example social networks (AAR ’ 08)

Summary and Discussion

- Test paths are a useful tool for learning circuits!
- Learnability (once again) surprisingly sensitive to small changes in parameters.
- We have an interesting model in which we made progress in learning probabilistic circuits/ Bayesian Networks

Results Table

Circuit	Fan-in	Topology	Gates	VIQ Learnability
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Large ∑	constant	log depth	arbitrary	W(1) Hard in shortcut width
Large ∑	constant	bounded shortcut width	arbitrary	Poly-time
Analog	constant	bounded shortcut width	arbitrary	Poly-time approximate
Probabilistic Boolean	constant	log depth	arbitrary	Poly-time approximate
Probabilistic Large ∑	constant	log depth	arbitrary	Poly-time w/ FIQs ??? w/ VIQs
Probabilistic cyclic!	Unbounded	arbitrary	independent cascade	Poly-time w/ exact VIQs