Learning Social Networks, Actively and Passively

SFI Talk

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Yahoo! Research
(work done while at Yale University)

talk based on 2 papers, both with
Dana Angluin and James Aspnes

Learning Interaction Networks

- Reconstructing Evolutionary Trees via Distance Experiments
- Learning and Verifying Graphs (ie Genome Sequences with PCR)
- Learning Large-Alphabet Circuits (ie Gene Regulatory Networks)
- Learning Bayesian Networks
- Actively Learning Social Networks
- Passively Learning Social Networks

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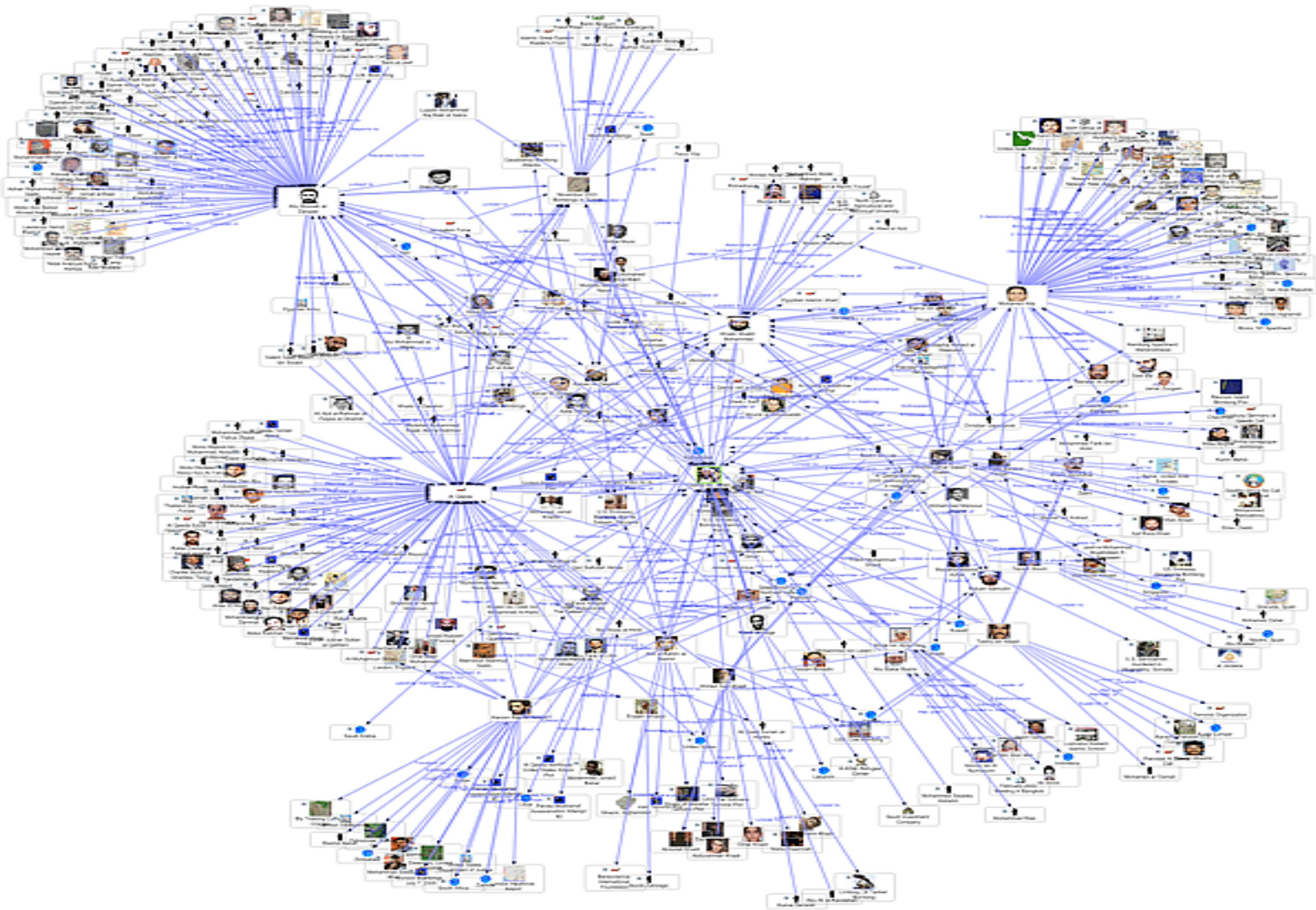
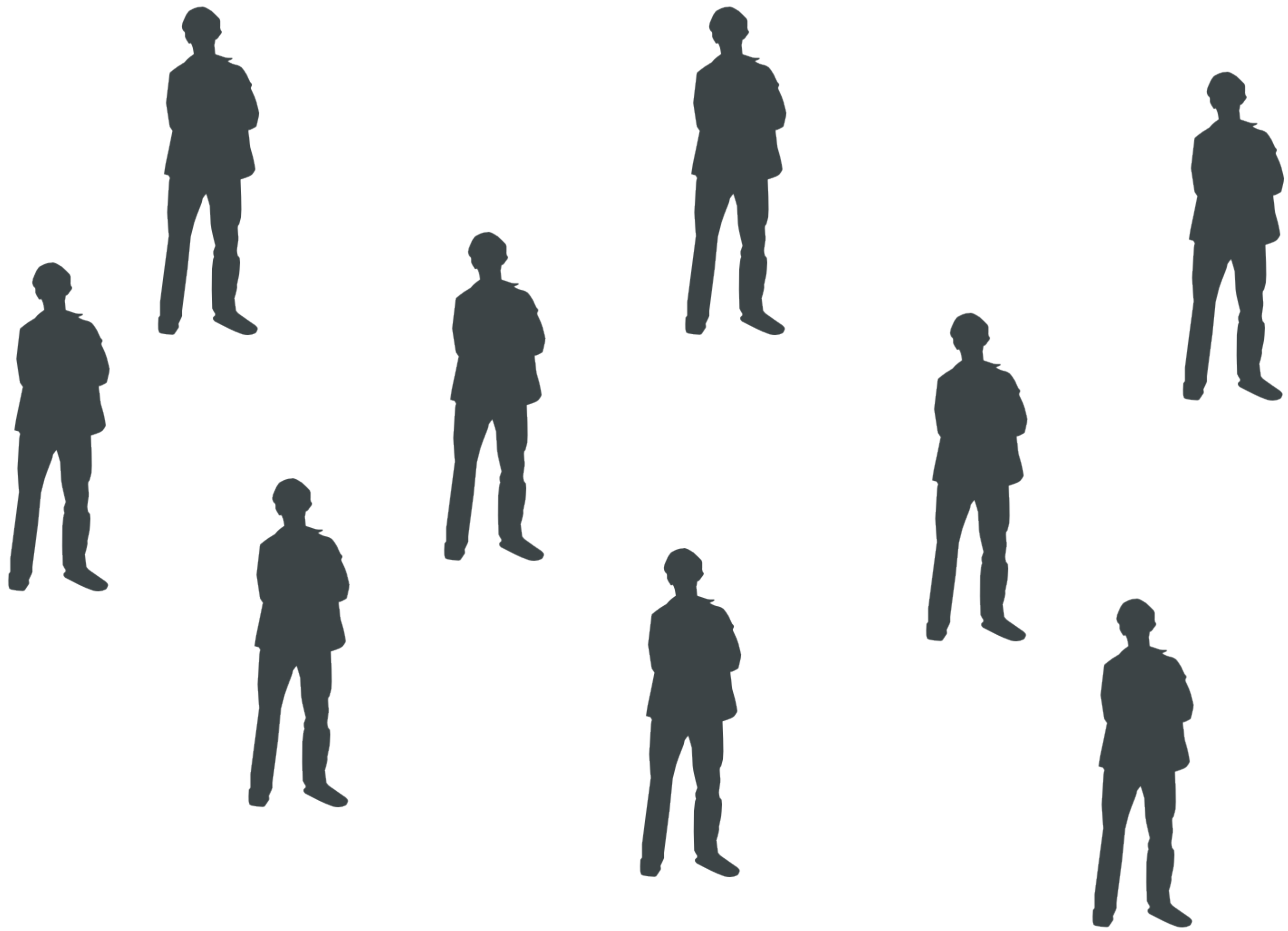
- Actively Learning Social Networks
- Passively Learning Social Networks

How Do We Learn Social Networks?



Trends Spreading through a Social Network



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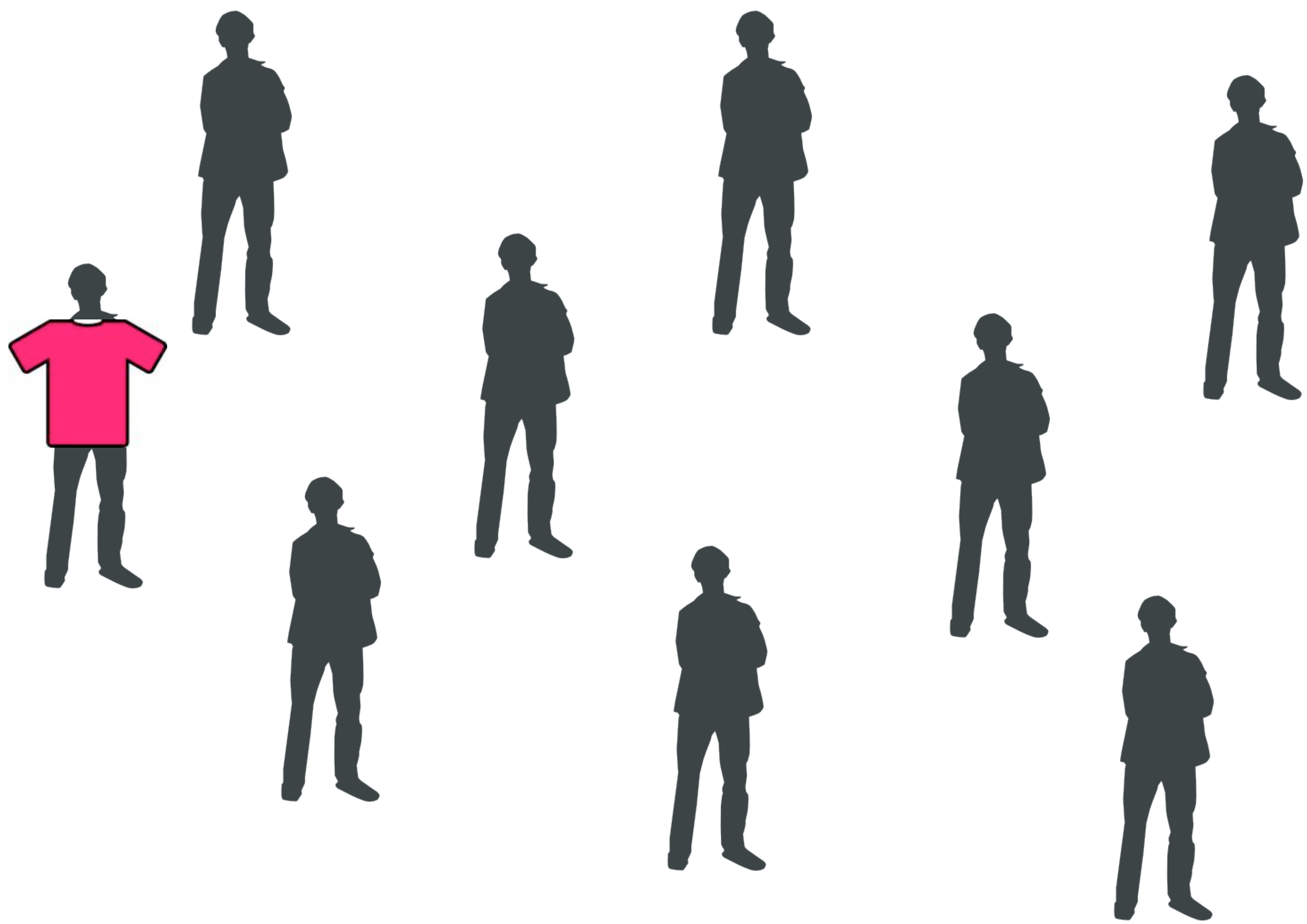

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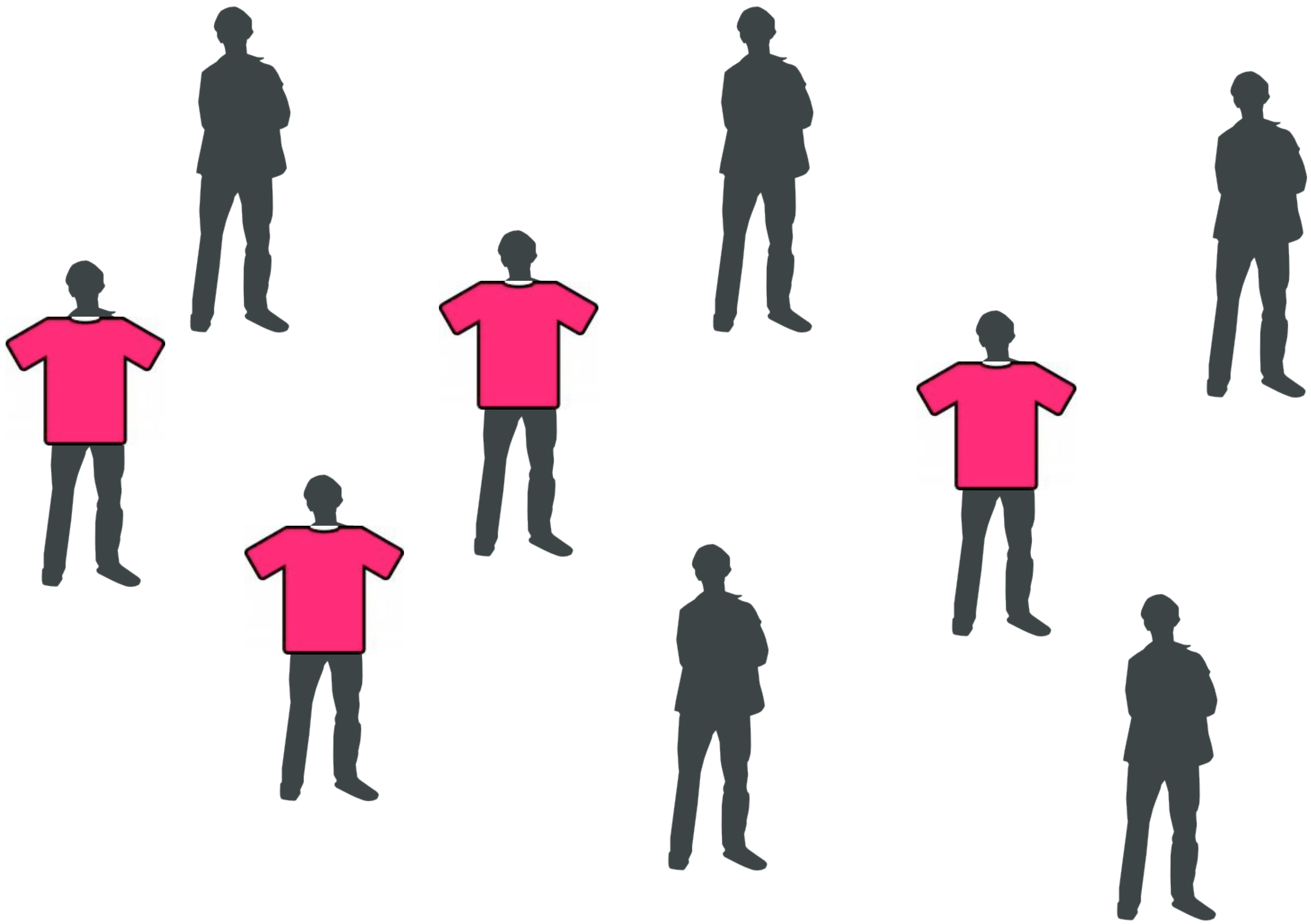

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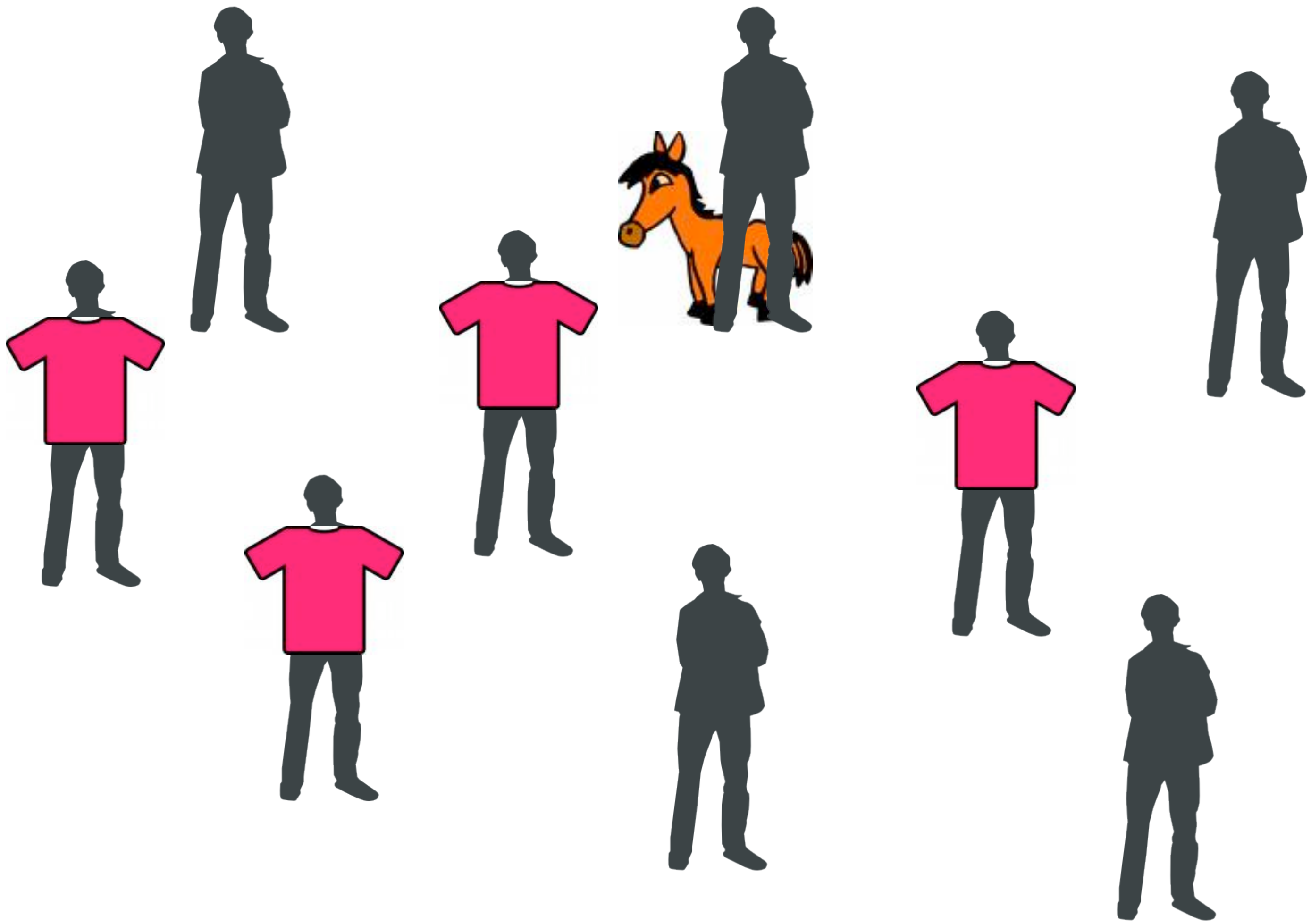

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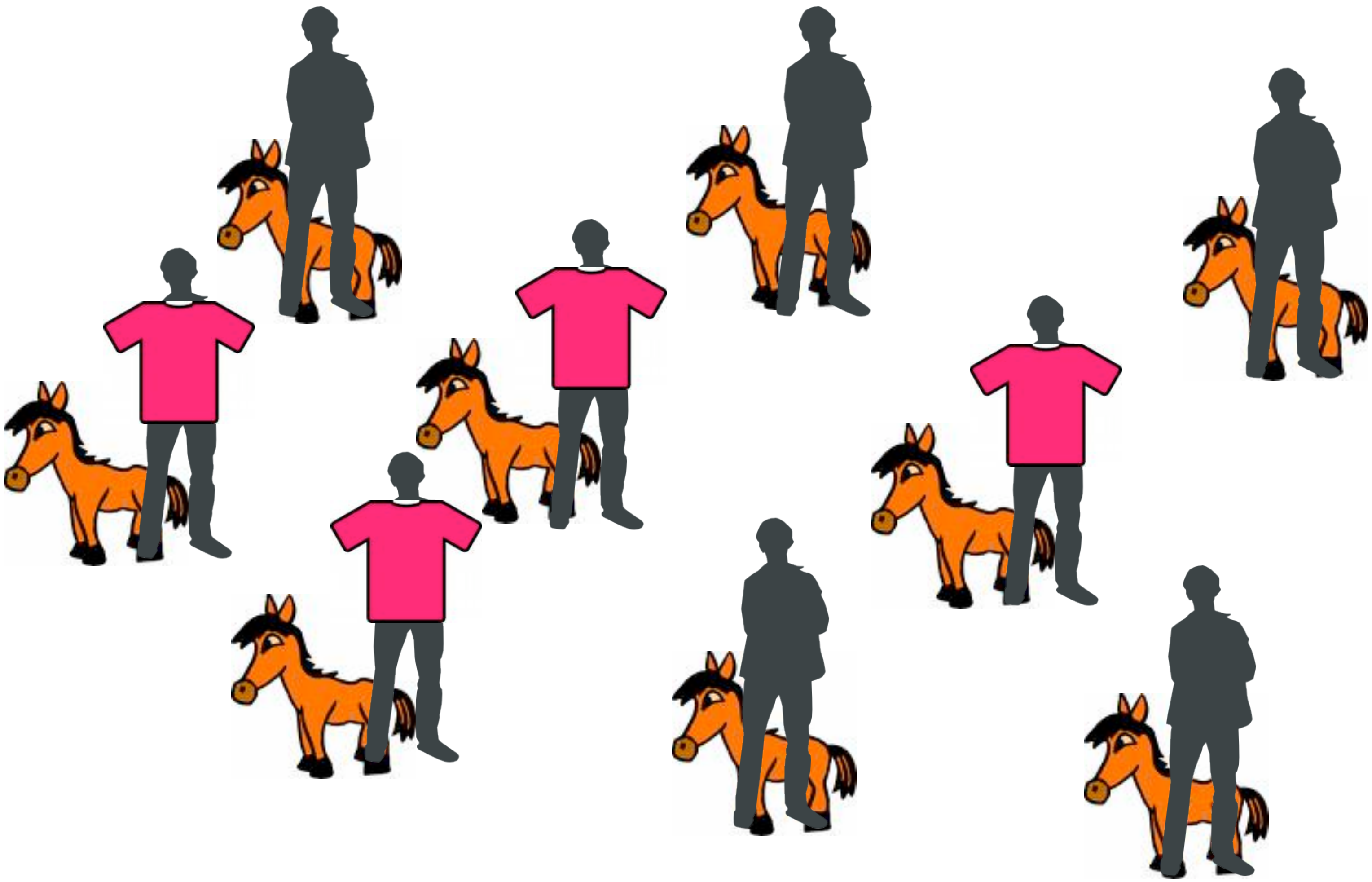
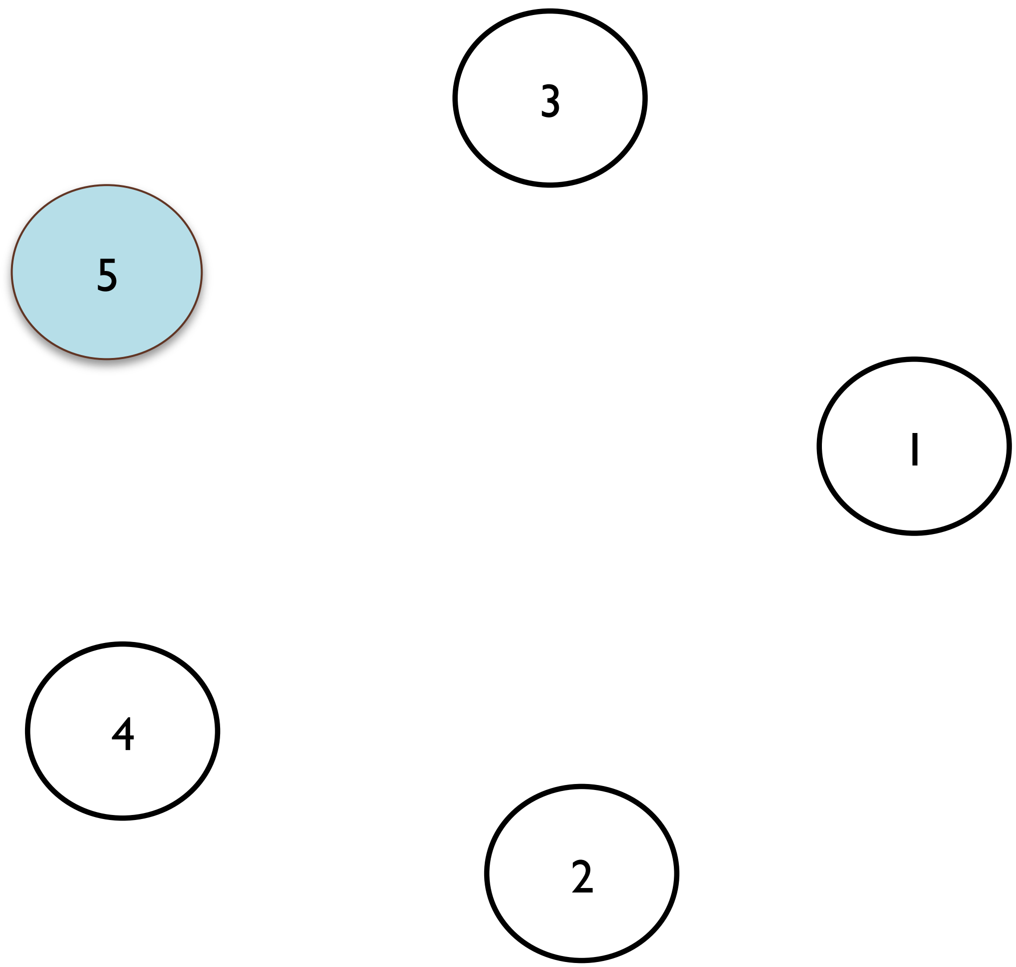


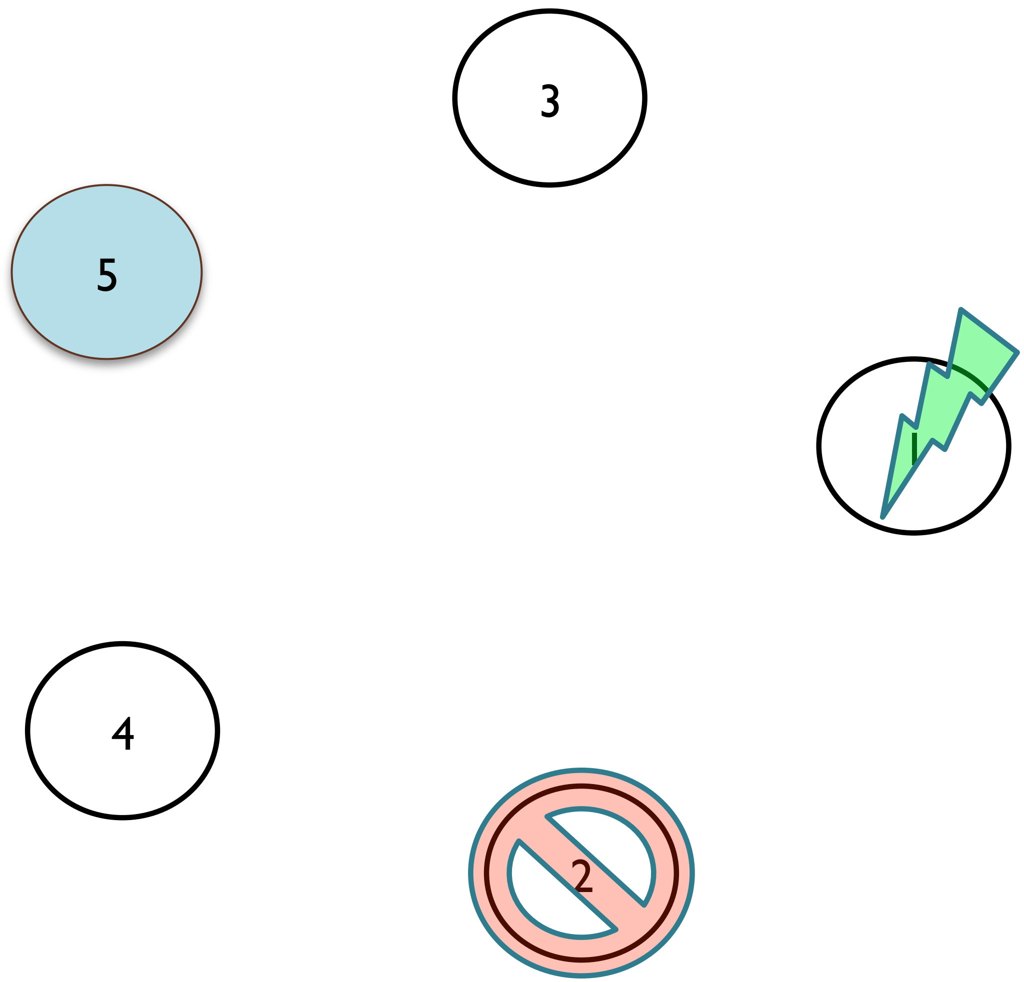
Value Injection Queries (VIQs) an Overview

- Model to study learning hidden circuits [AACW '06] – inspired by learning of gene regulatory networks.
- Allows for perturbing the circuit anywhere, but observing only its one output (ie phenotype, vote, $).
- In between input/output models and fully observable models.
- We apply VIQs to learning independent cascade social networks.

What the Learner Sees



Activations and Suppressions



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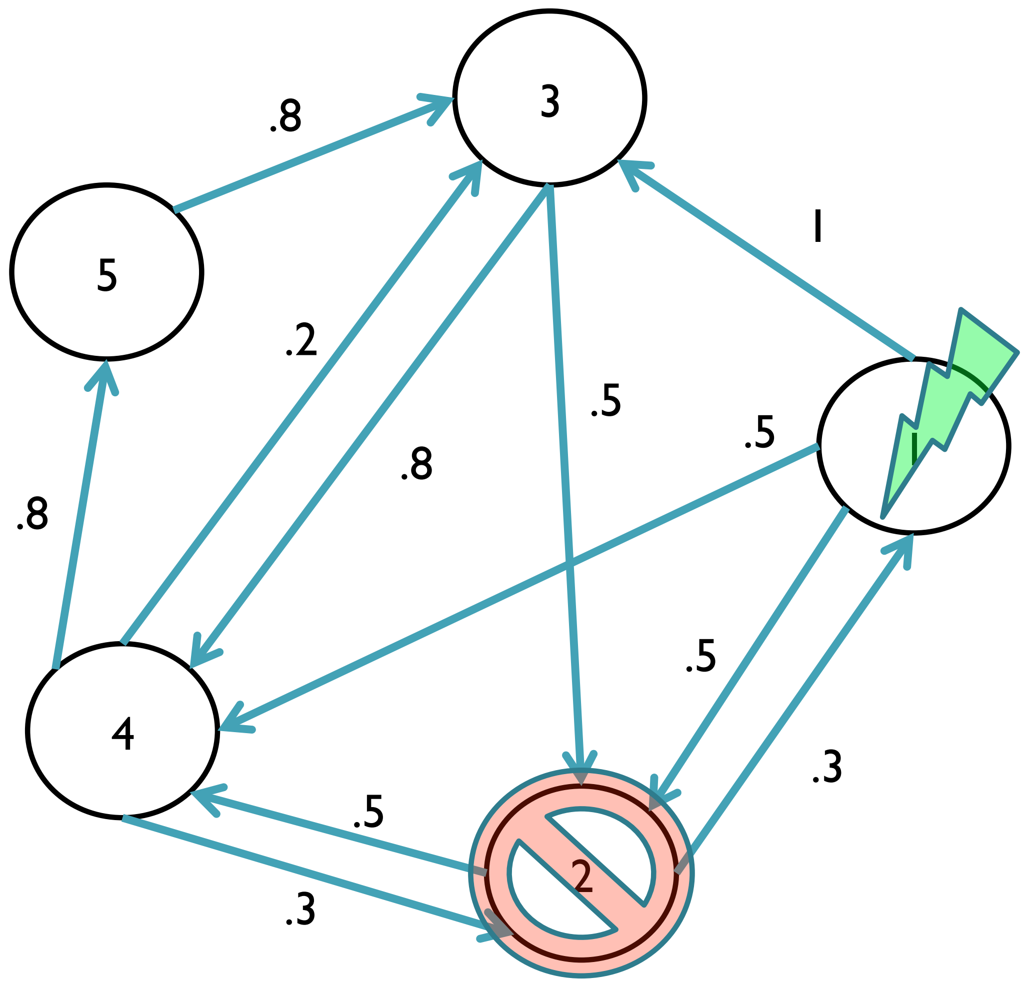

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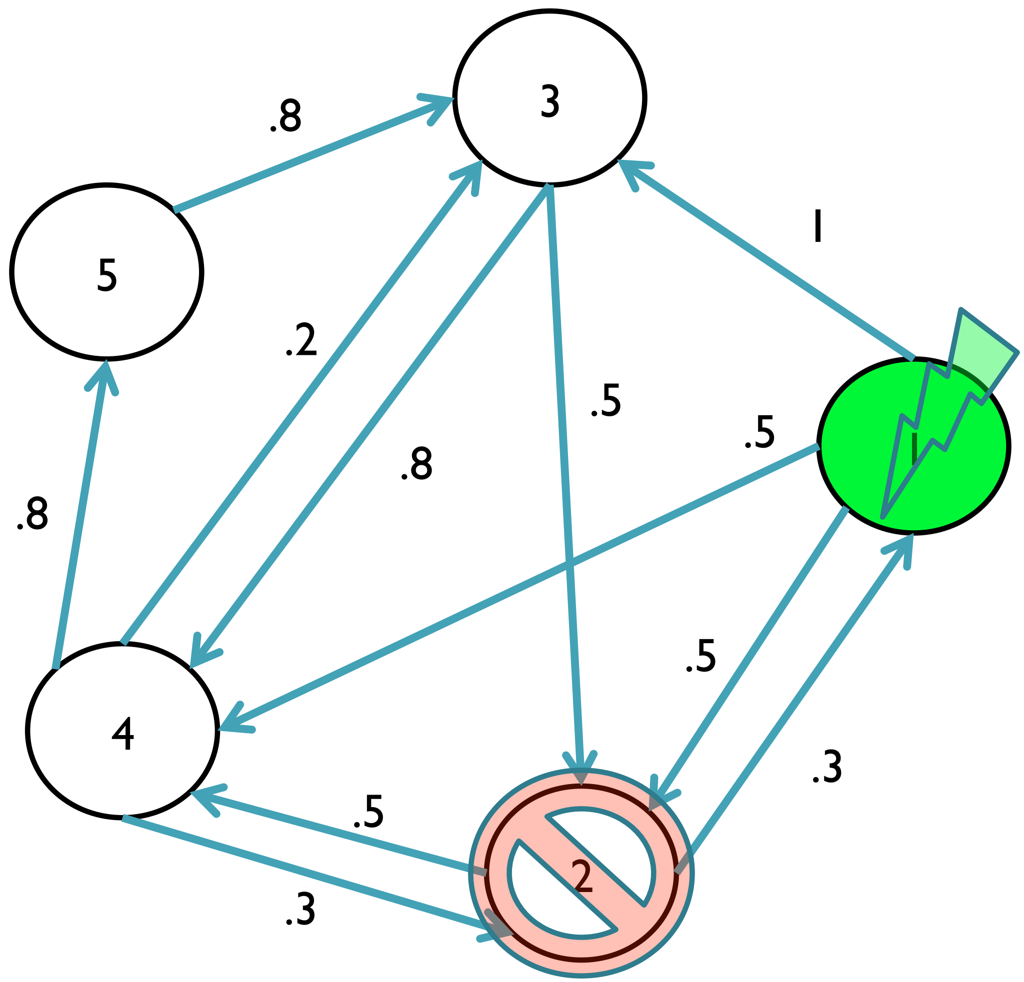

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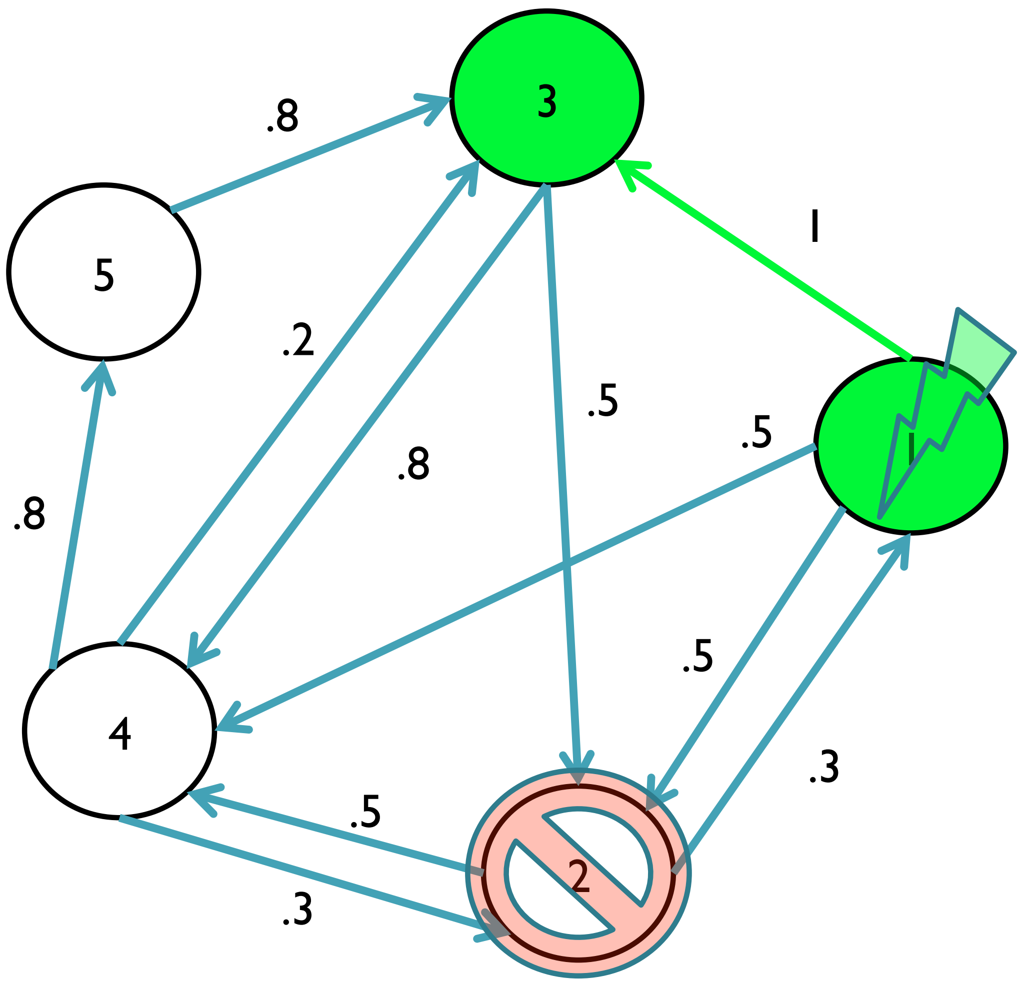

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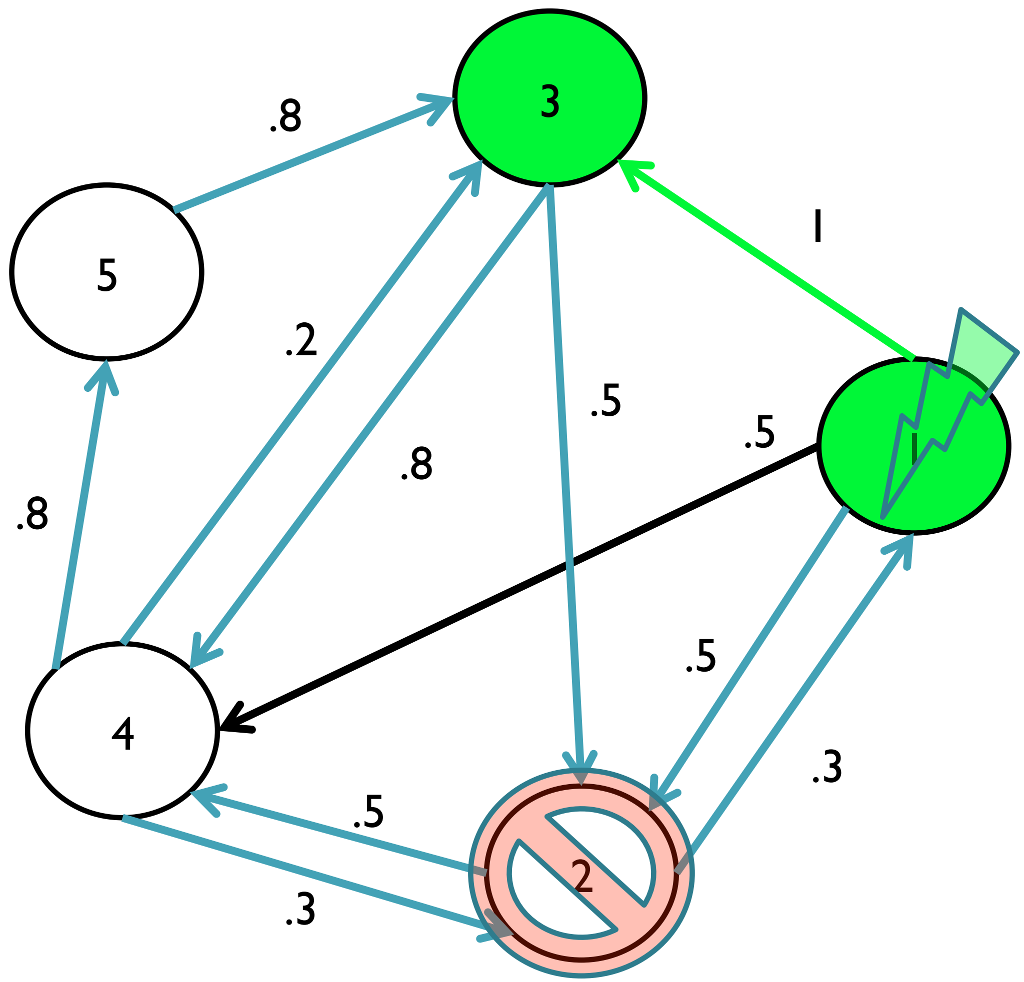

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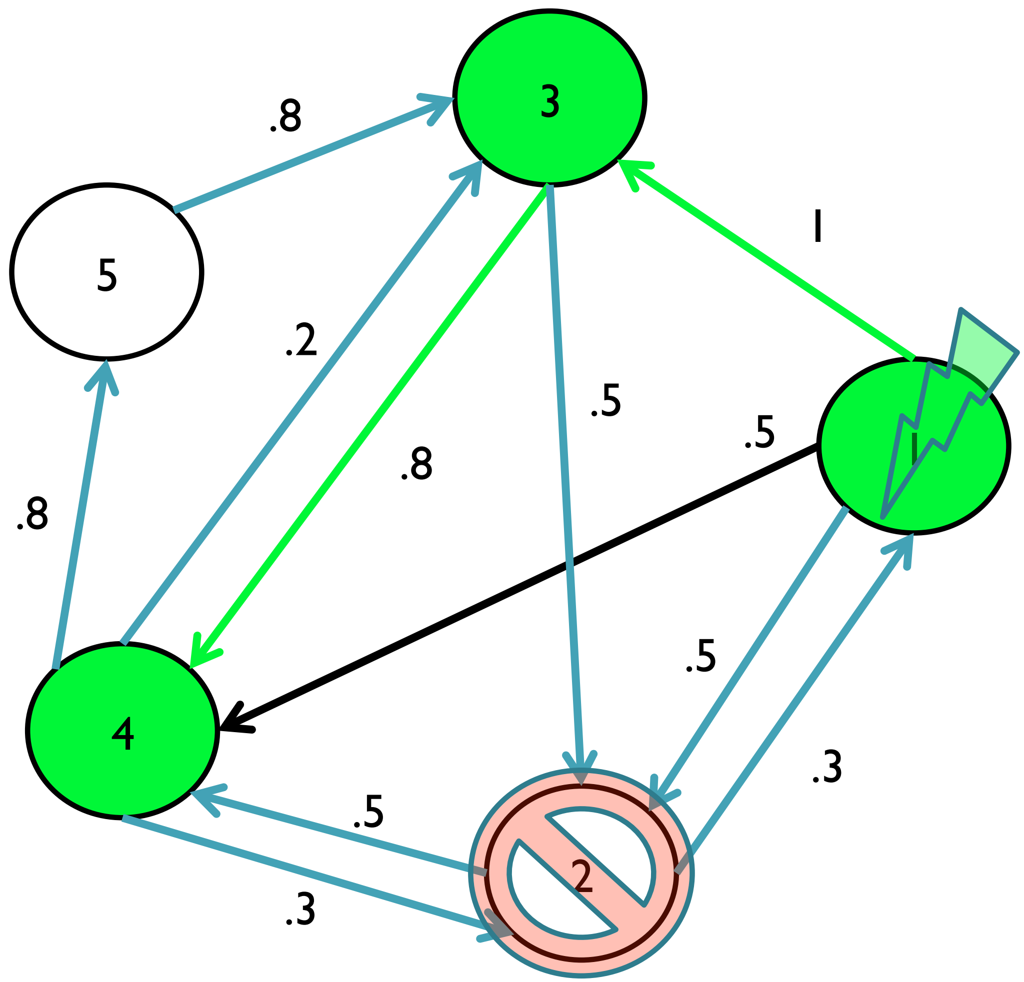

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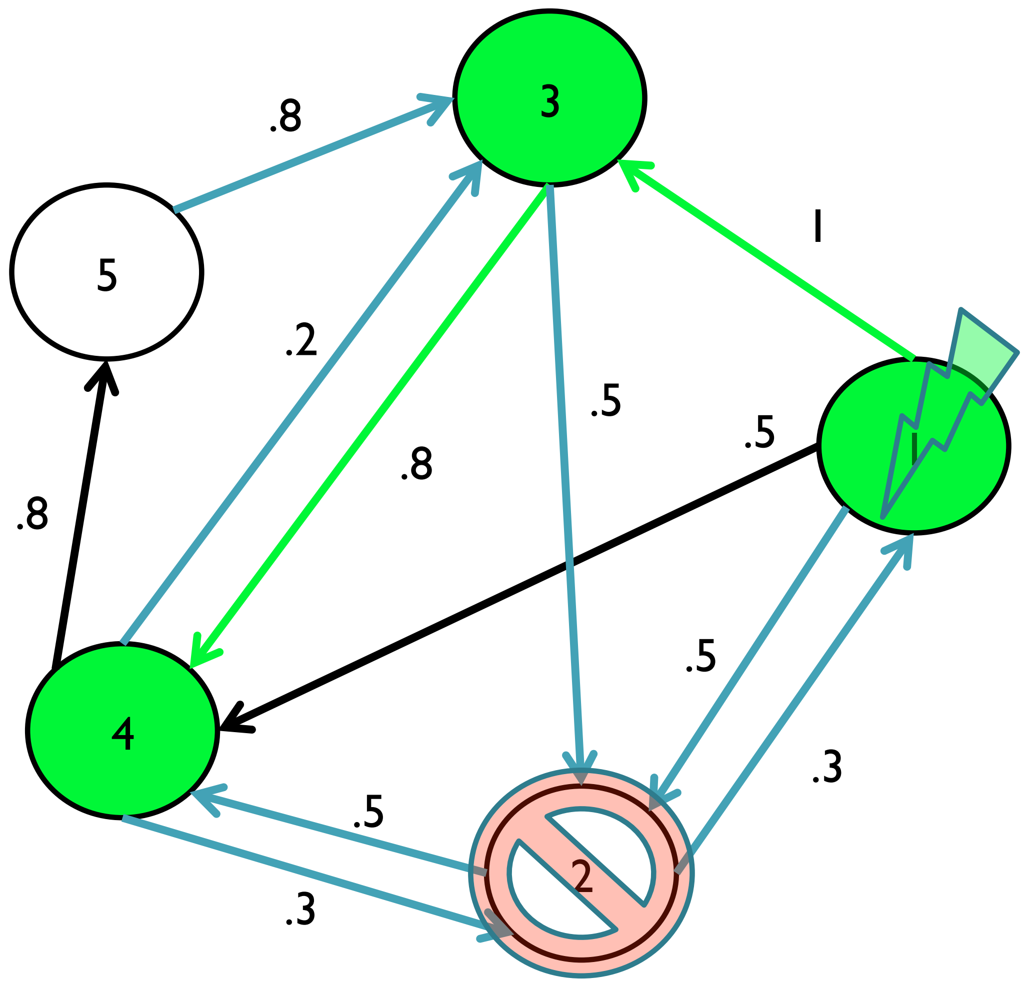

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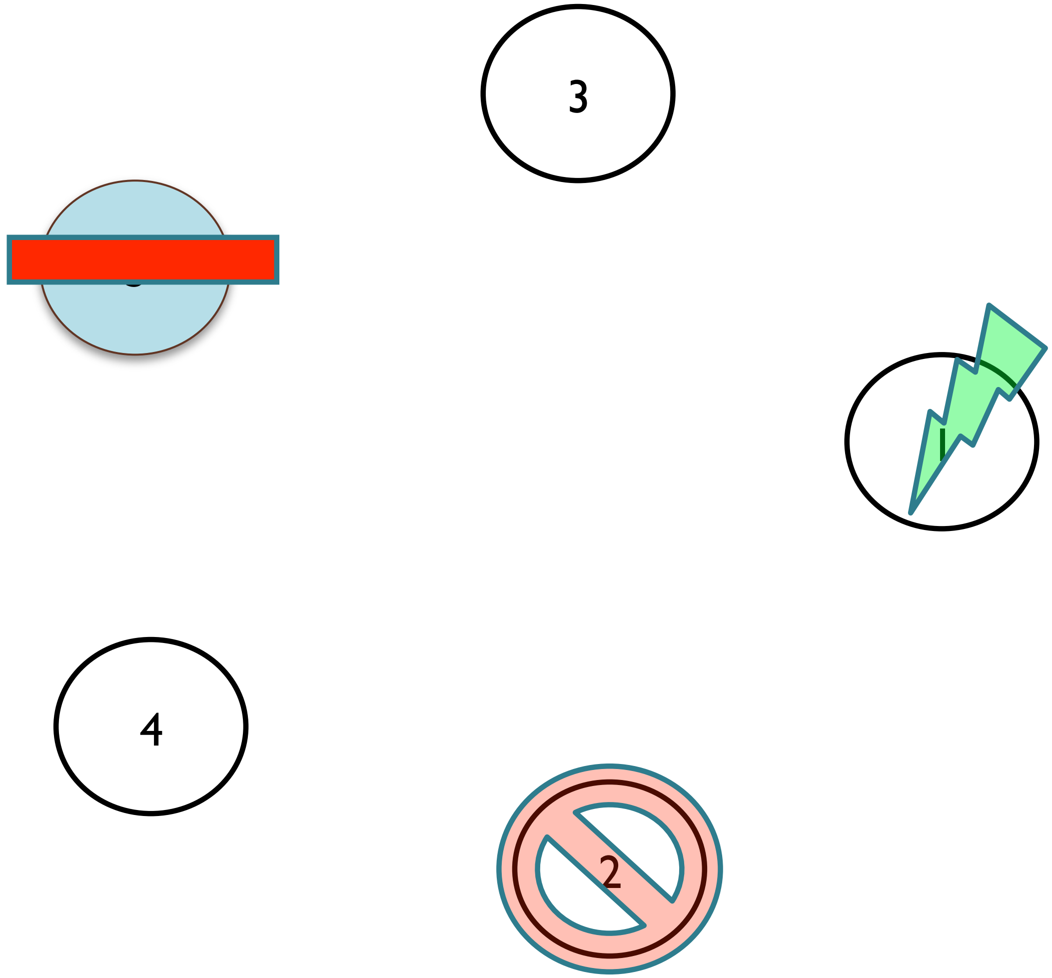

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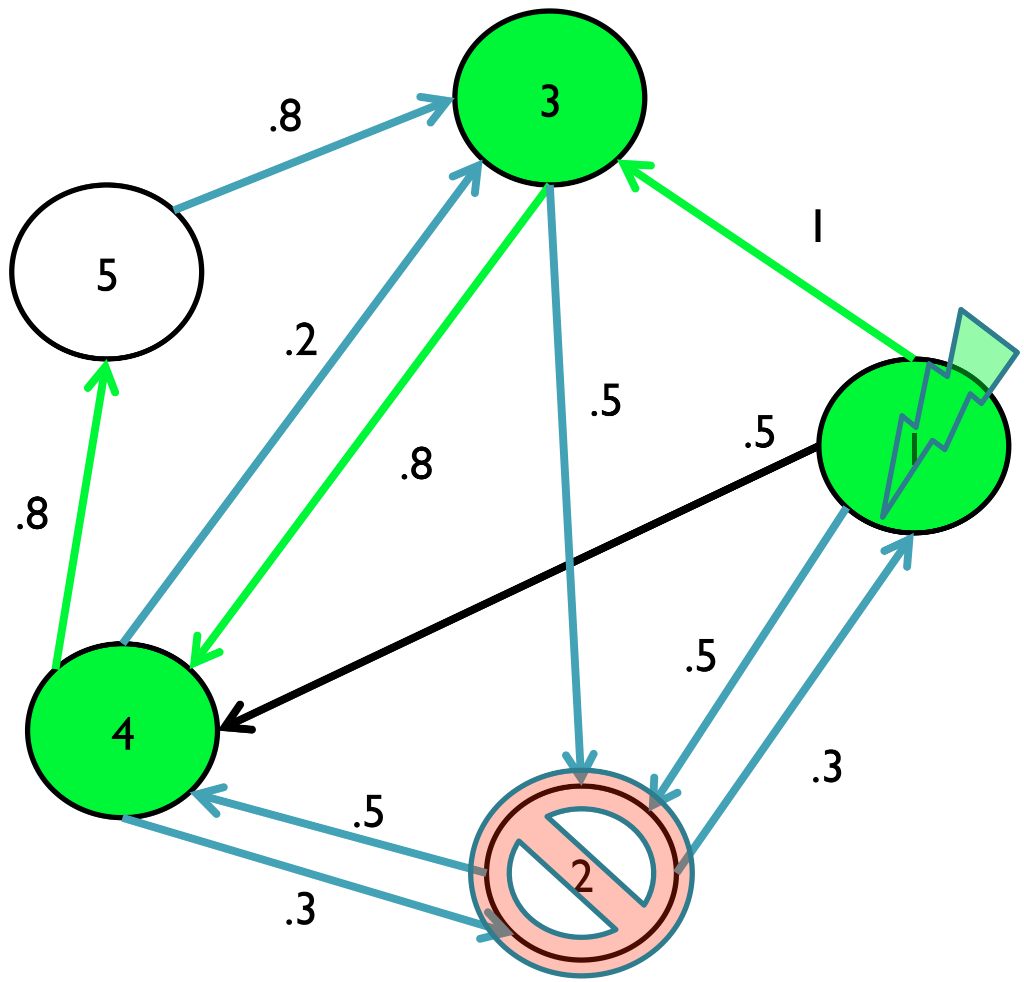

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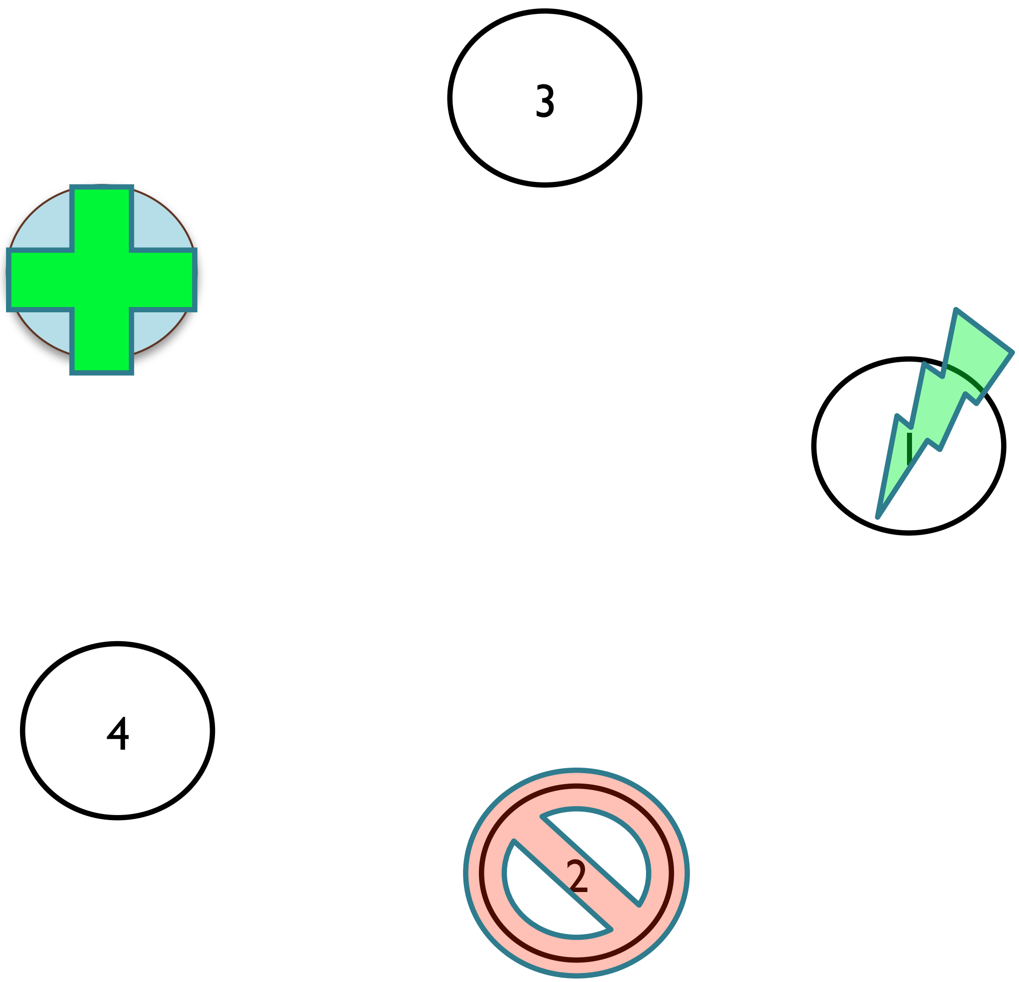
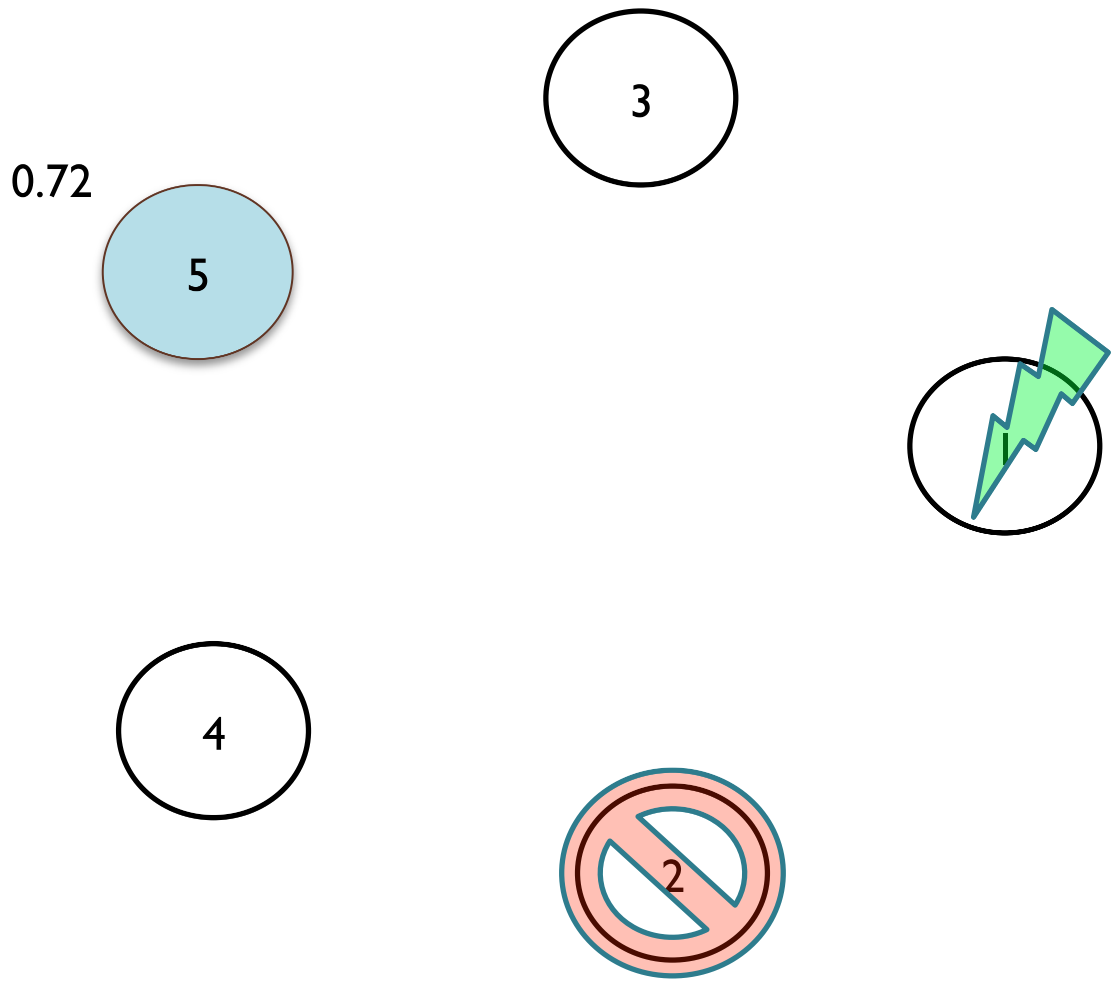


Exact Value Injection Queries



The Learning Task

- Two social networks S and S' are behaviorally equivalent if for any experiment e, S(e) = S'(e)

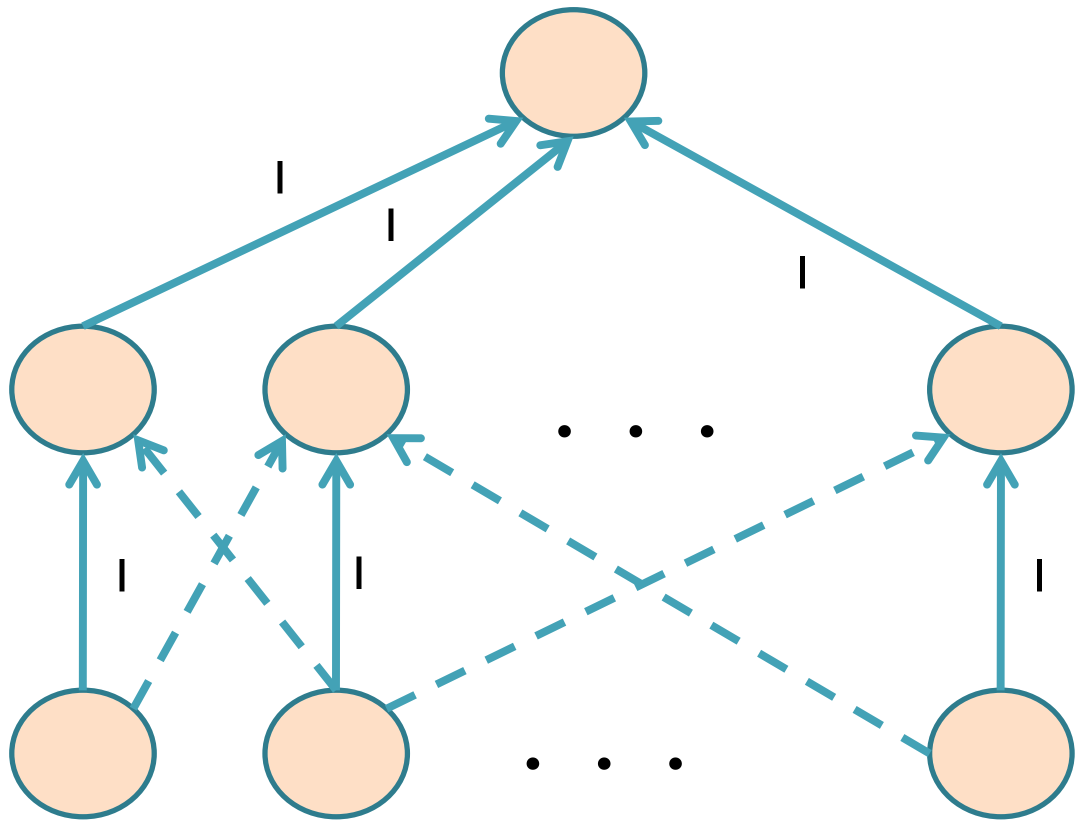
- Given access to a hidden social network S*, the learning problem is to find a social network S behaviorally equivalent to S* using value injection queries.

The Percolation Model

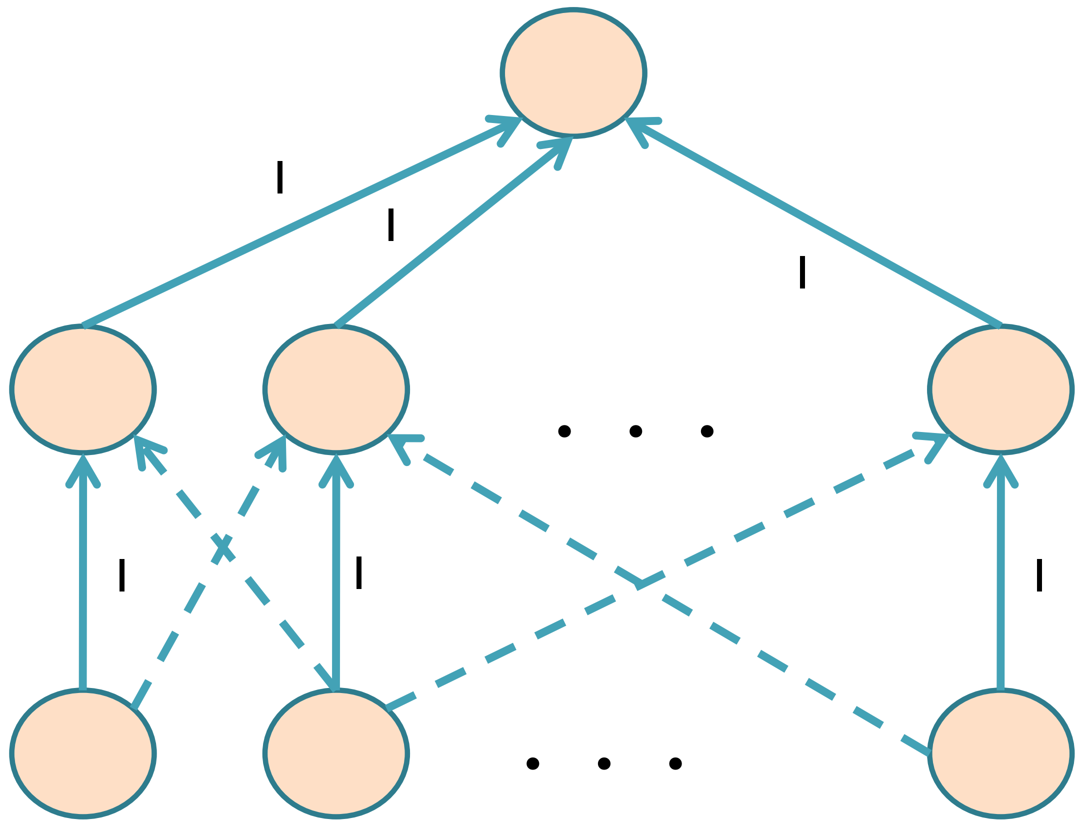
Given a network S and a VIQ

- All edges entering or leaving a suppressed node are automatically "closed."
- Each remaining edge (u,v) is "open" with probability $p_{(u,v)}$ and "closed" with probability $(1 - p_{(u,v)})$
- The result of a VIQ is the probability there is a path from a activated node to the output via open edges in S

A Lower Bound

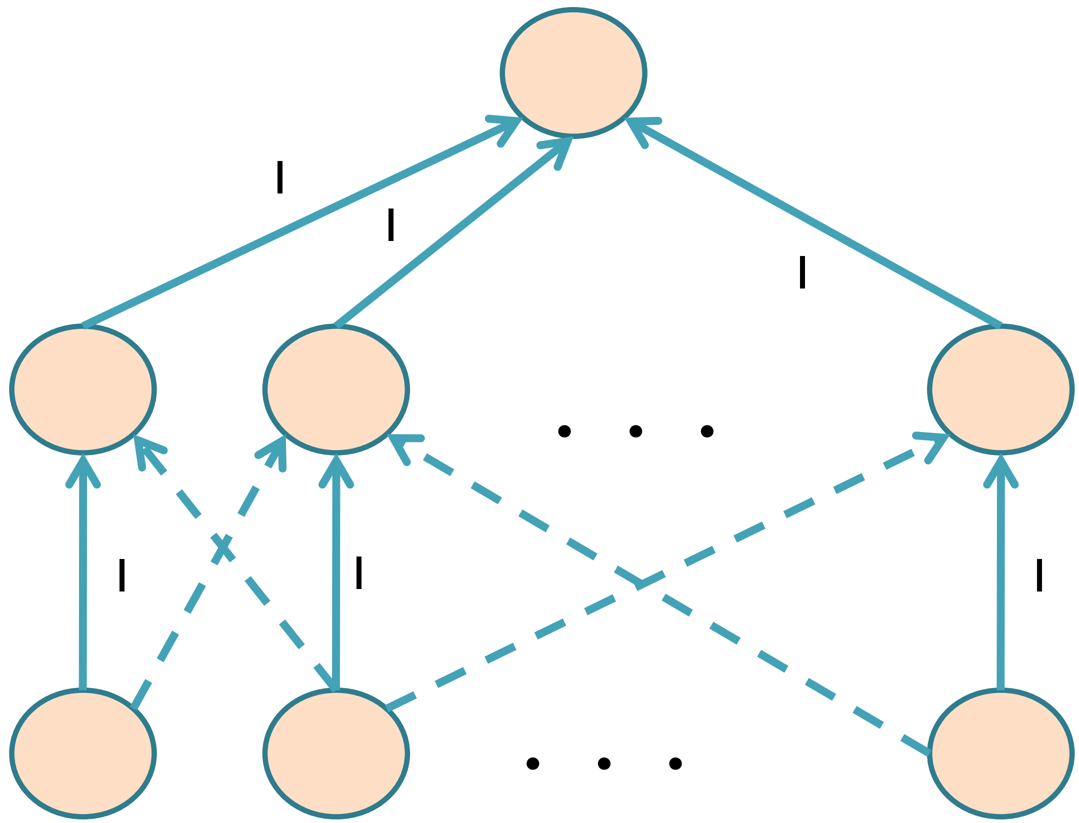


A Lower Bound



All queries give 1-bit answers

A Lower Bound



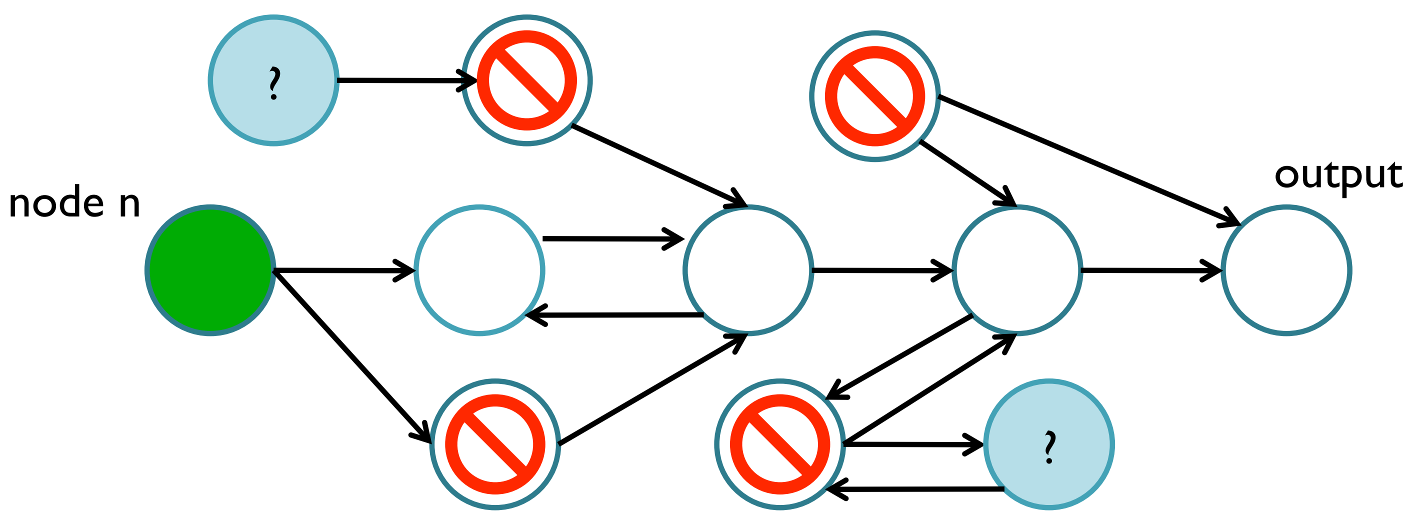
$2^{\Omega(n^2)}$ such graphs, $\Omega(n^2)$ l.b.

An Algorithm: First Some Definitions

- The depth of a node is its distance to the root
- An Up edge is an edge from a node of larger depth to a node of smaller depth
- A Level edge is an edge between two nodes of same depth
- A Down edge is an edge from a node at smaller depth to a node at higher depth
- A leveled graph of a social network is the graph of its Up edges

Excitation Paths

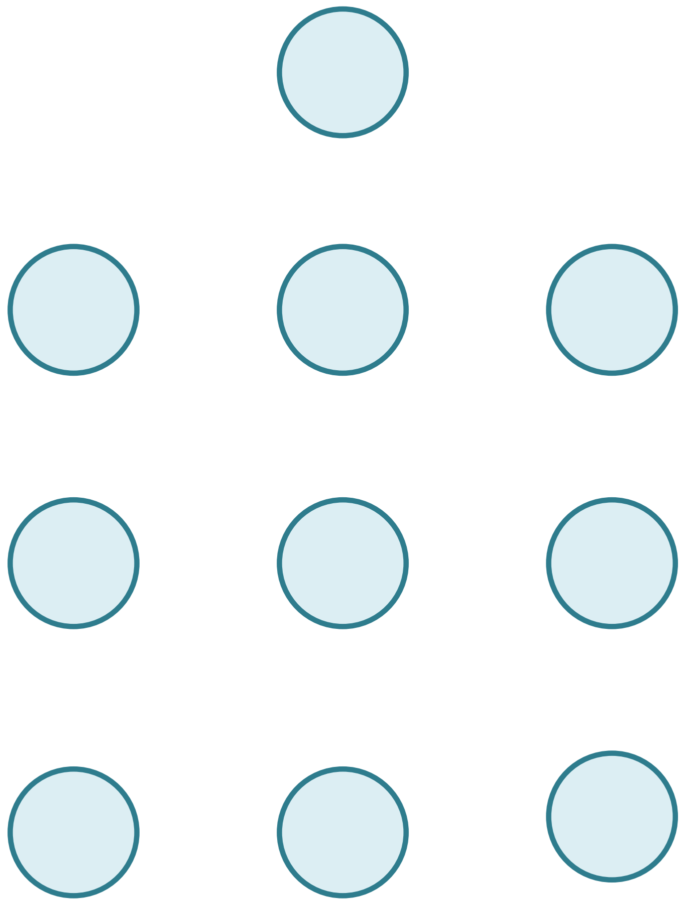
- An excitation path for a node n is a VIQ in which a subset of the free agents form a simple directed path from n to the output. All agents not on the path with inputs into the path are suppressed.
- We also have a shortest excitation path



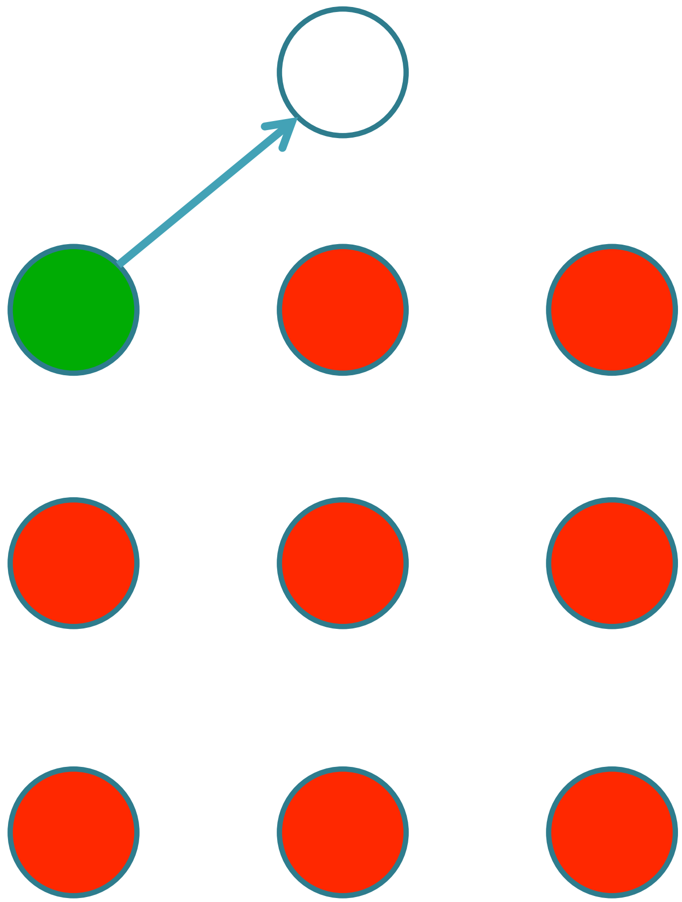
The Learning Algorithm For Networks w/o Probability1 Edges

- First Find-Up-Edges to learn the leveled graph of S
- For each level, Find-Level-Edges
- For each level, starting from the bottom, Find-Down-Edges

Find-Up-Edges



Find-Up-Edges



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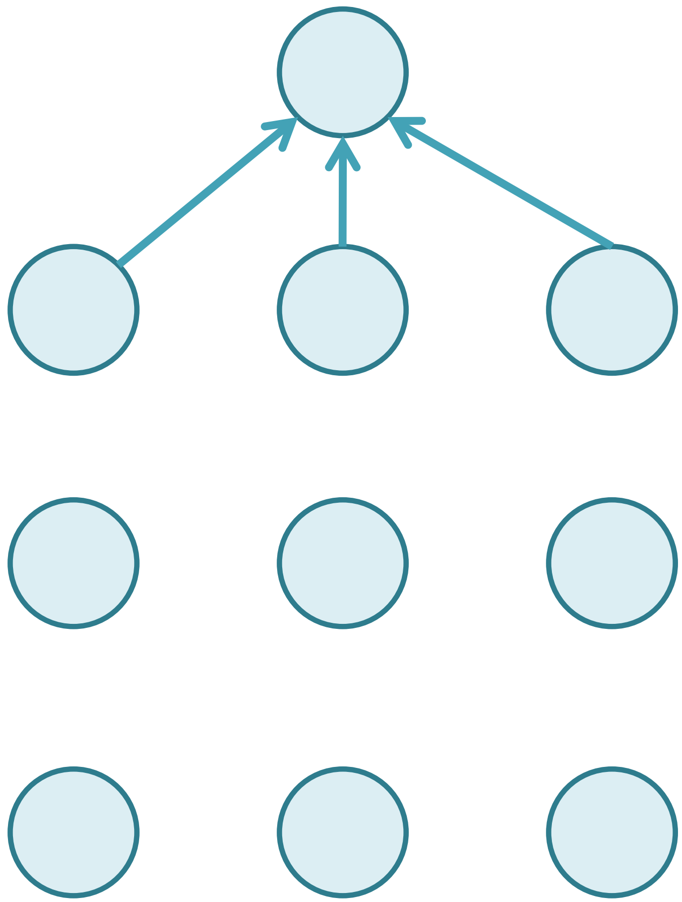

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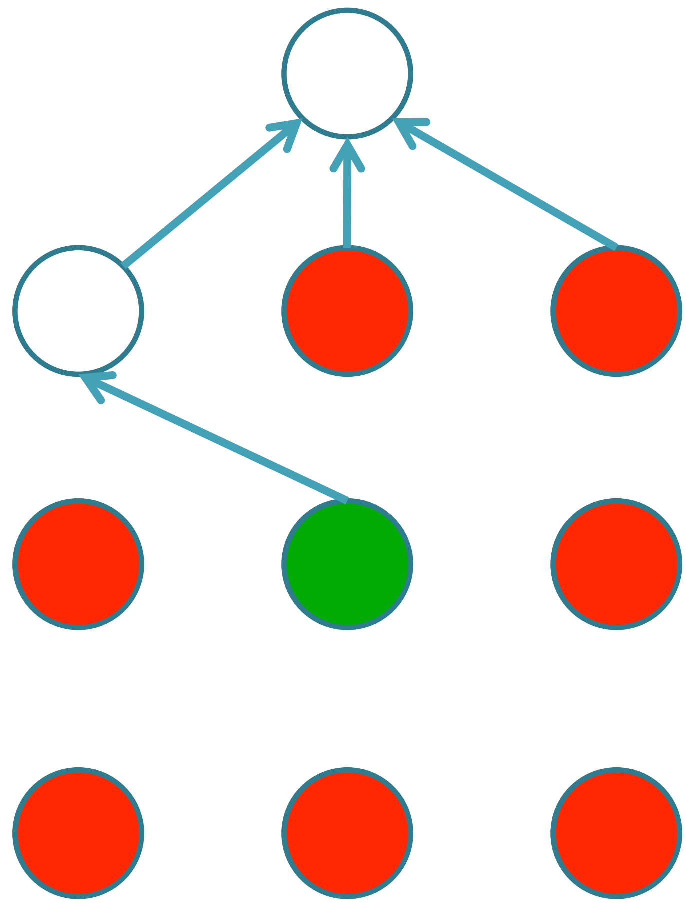

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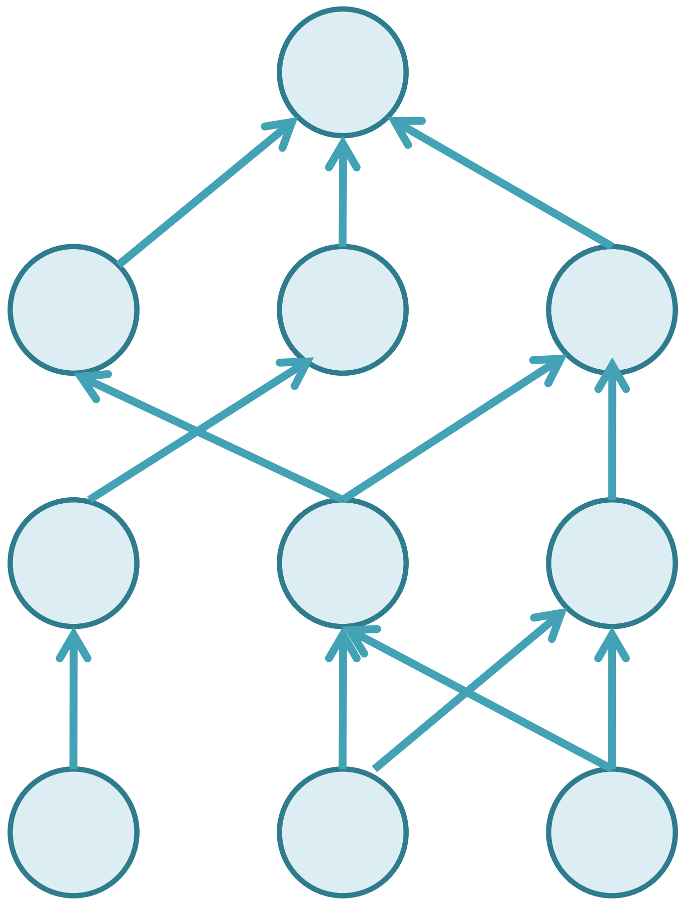

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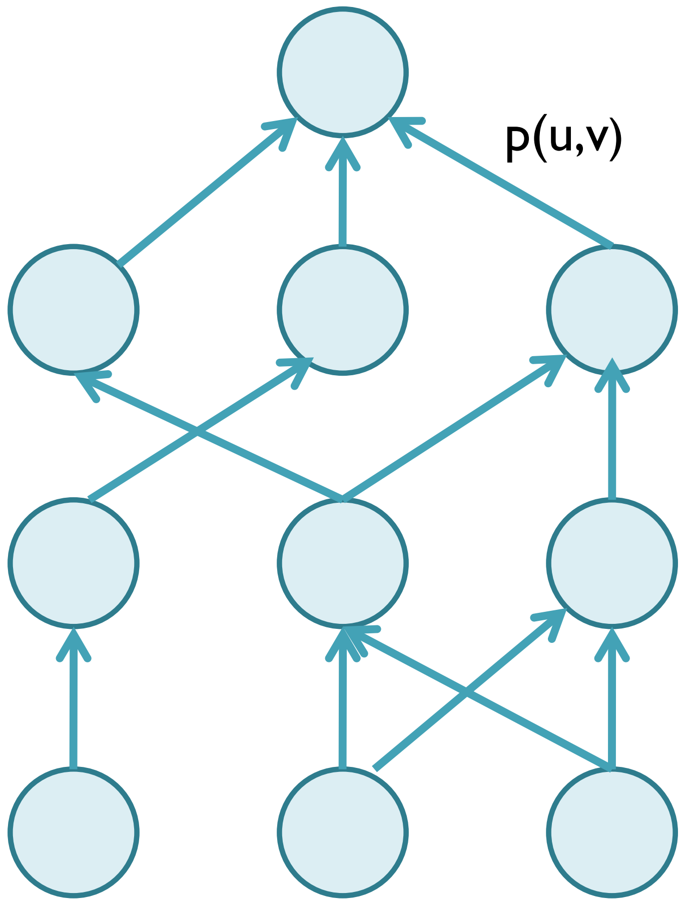
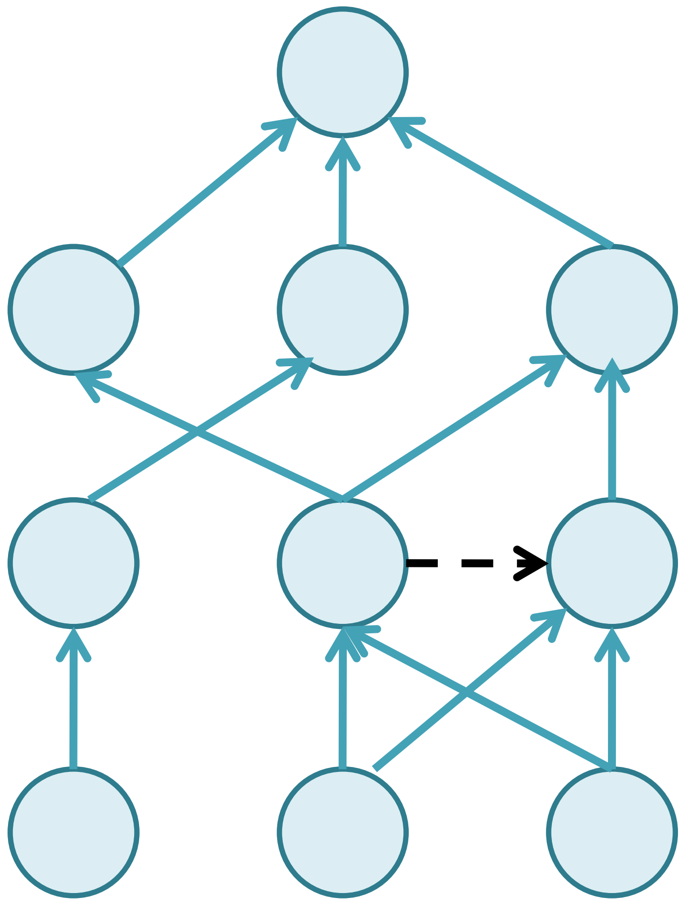


Find-Level-Edges



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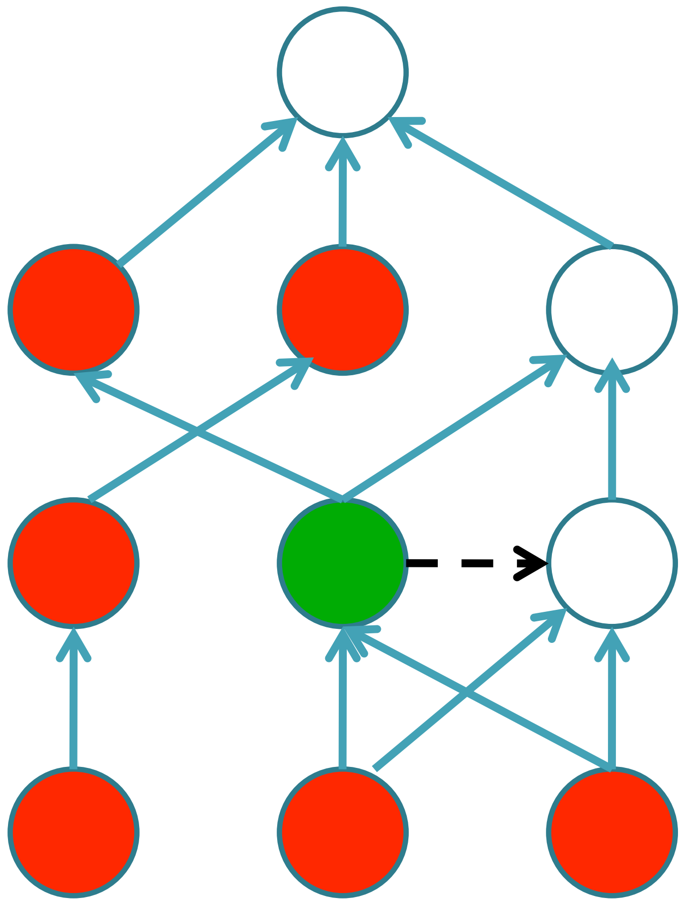

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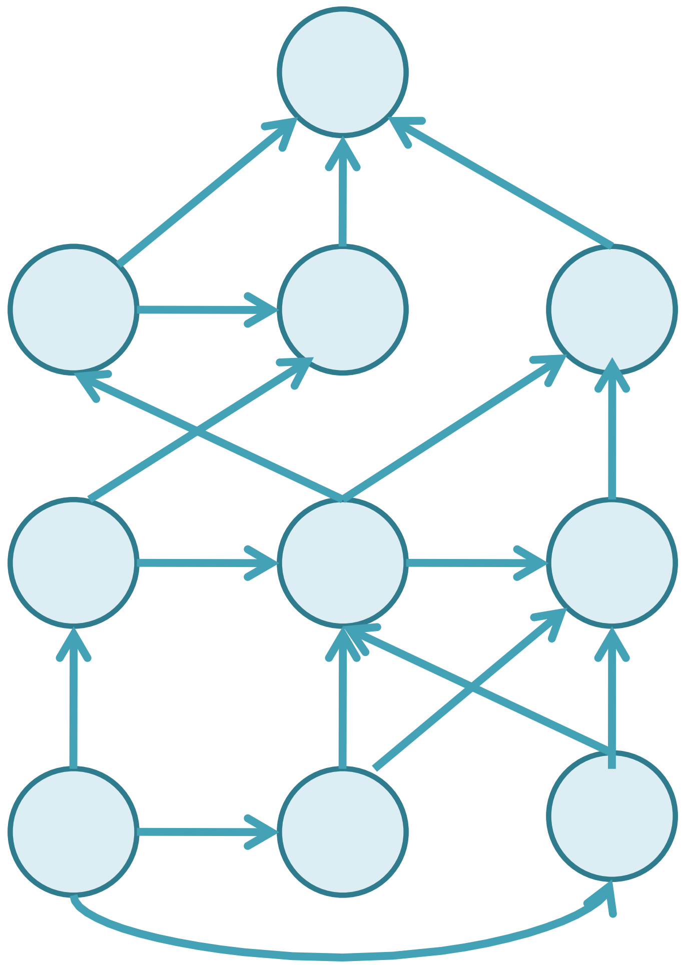
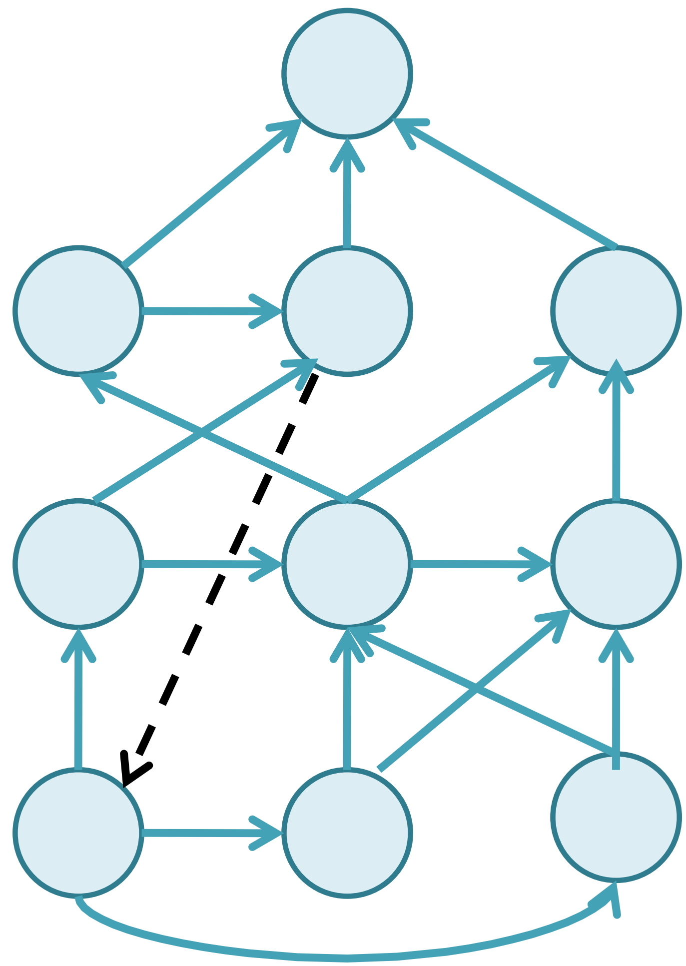


Find-Down-Edges



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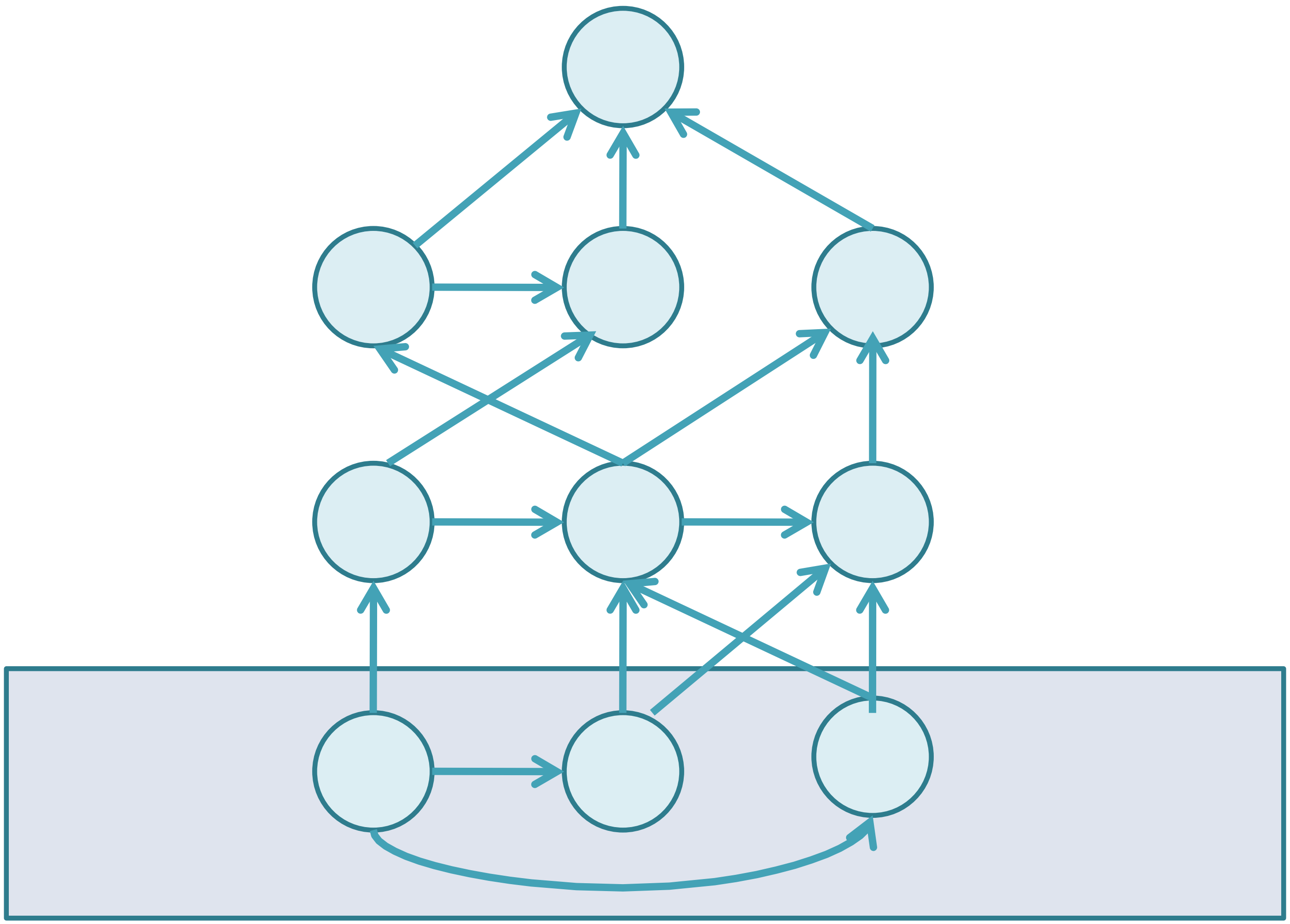

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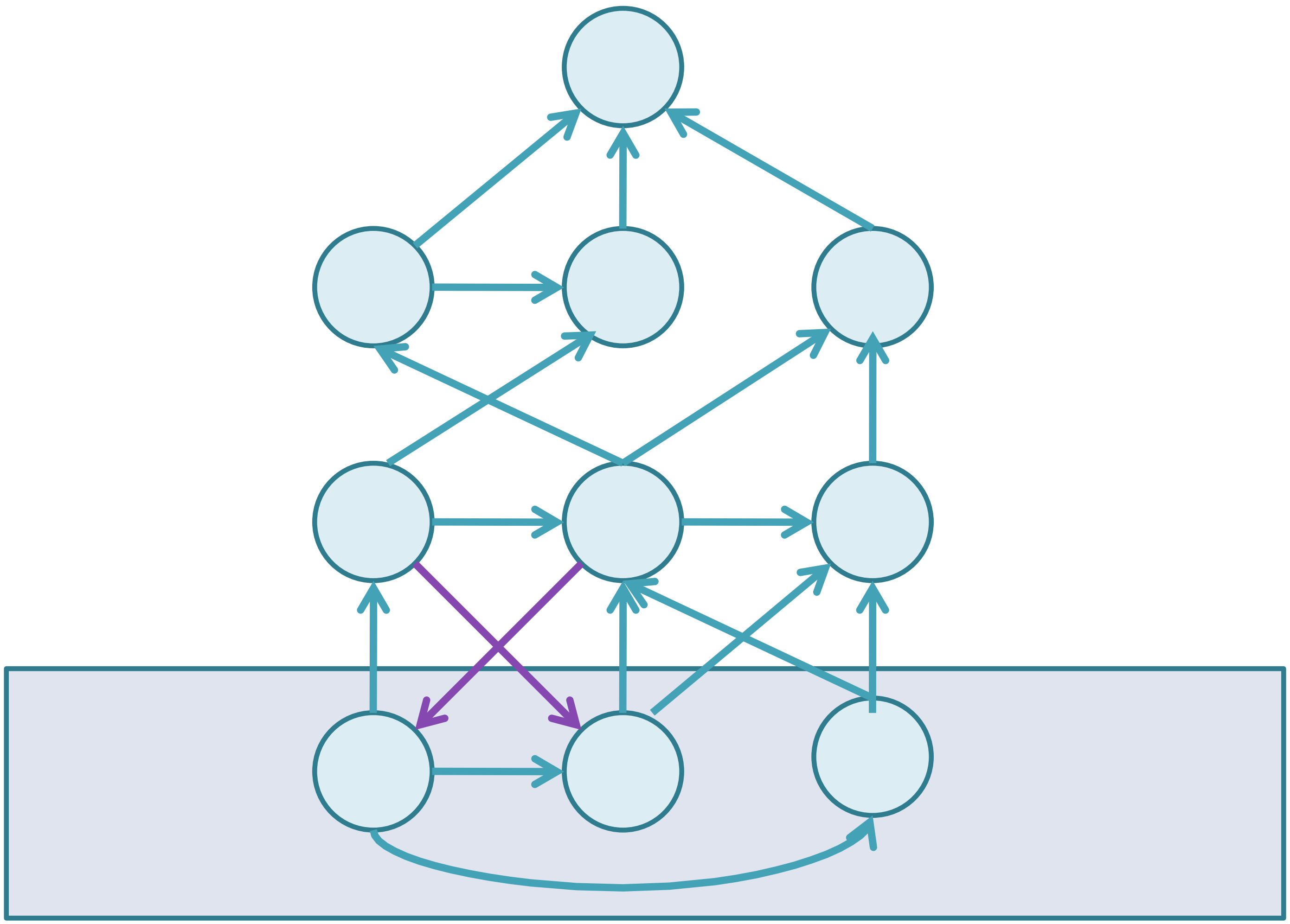

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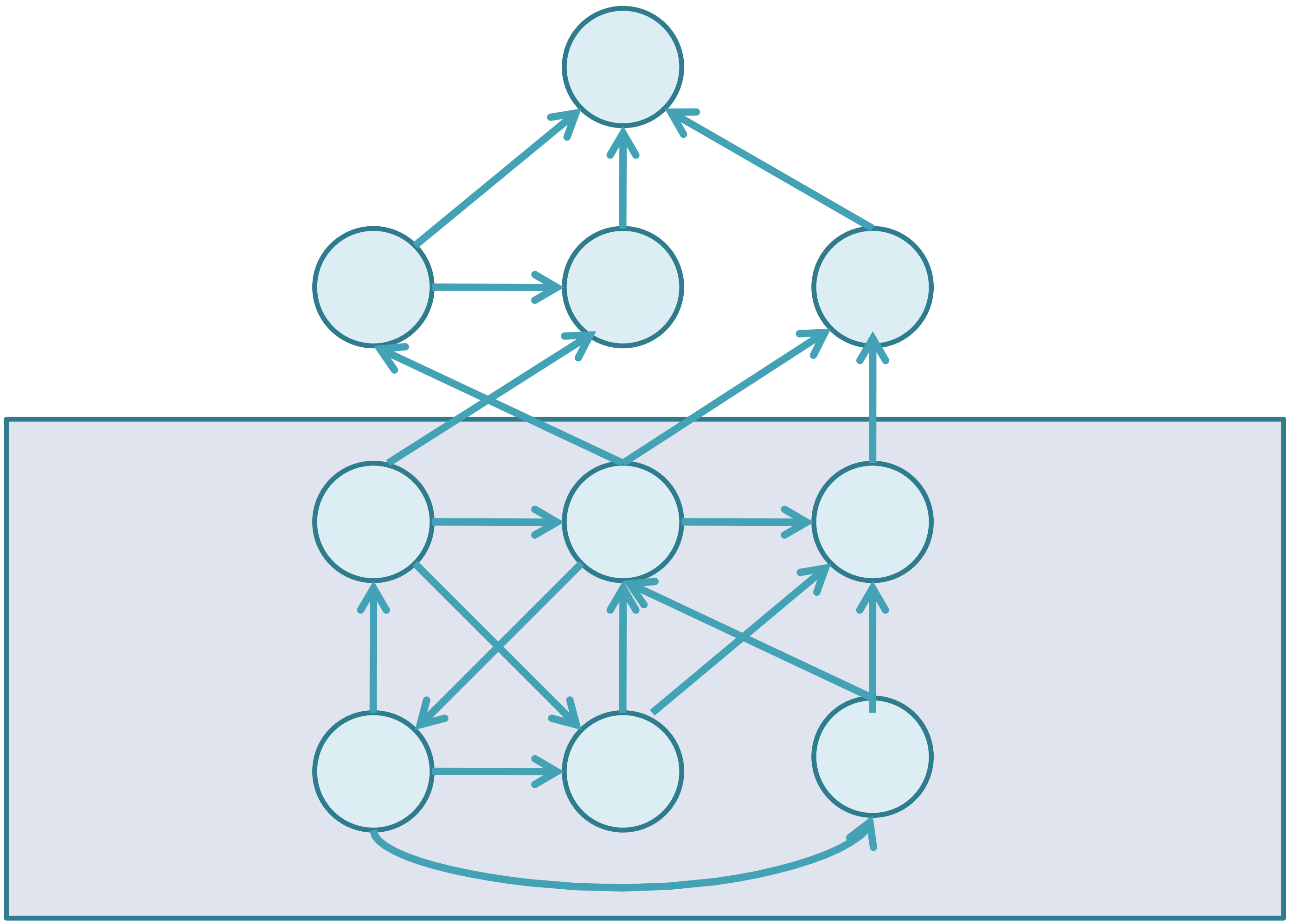

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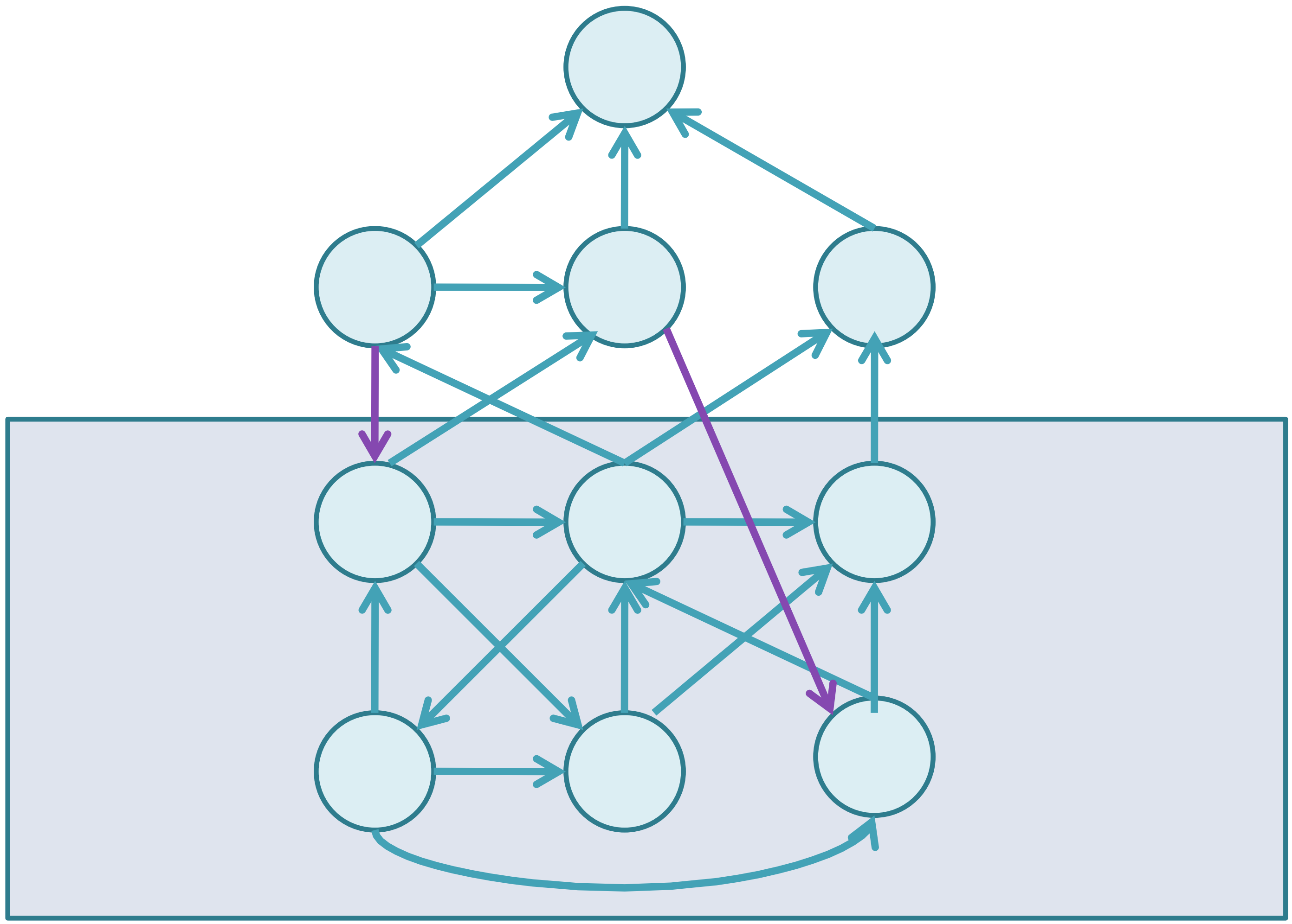

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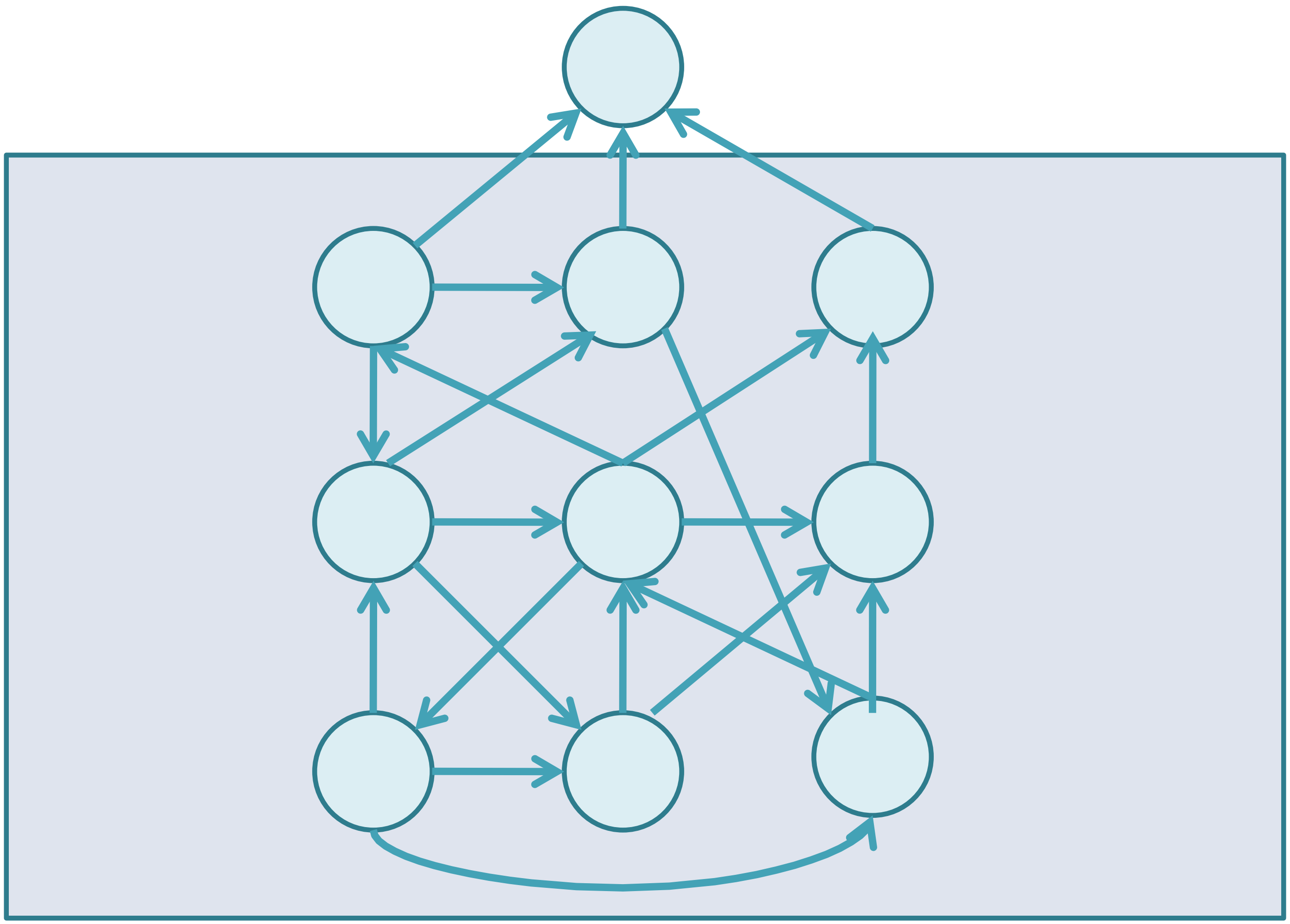

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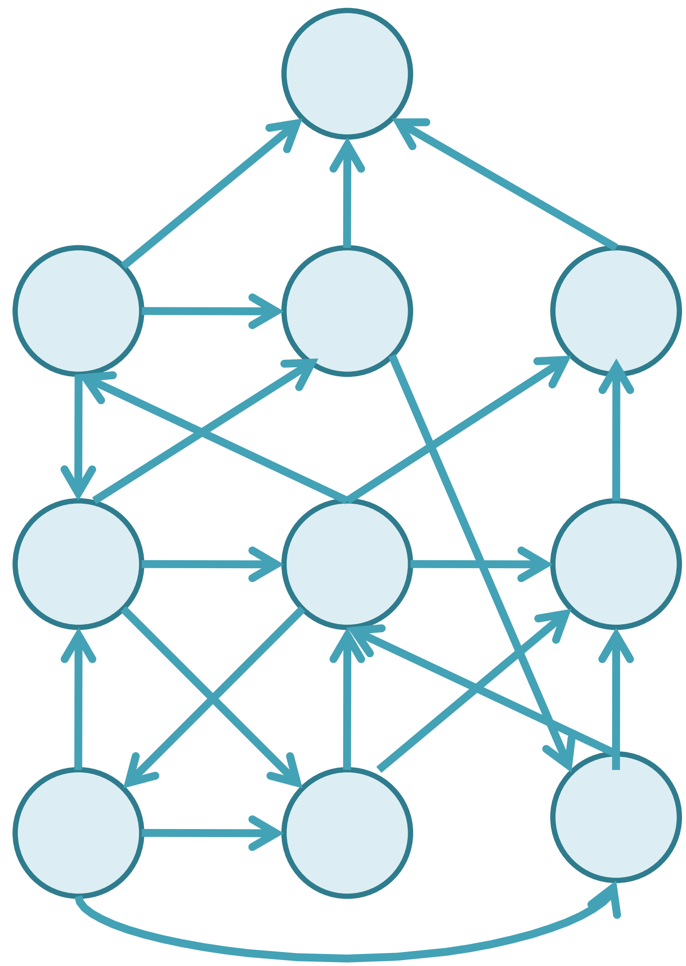
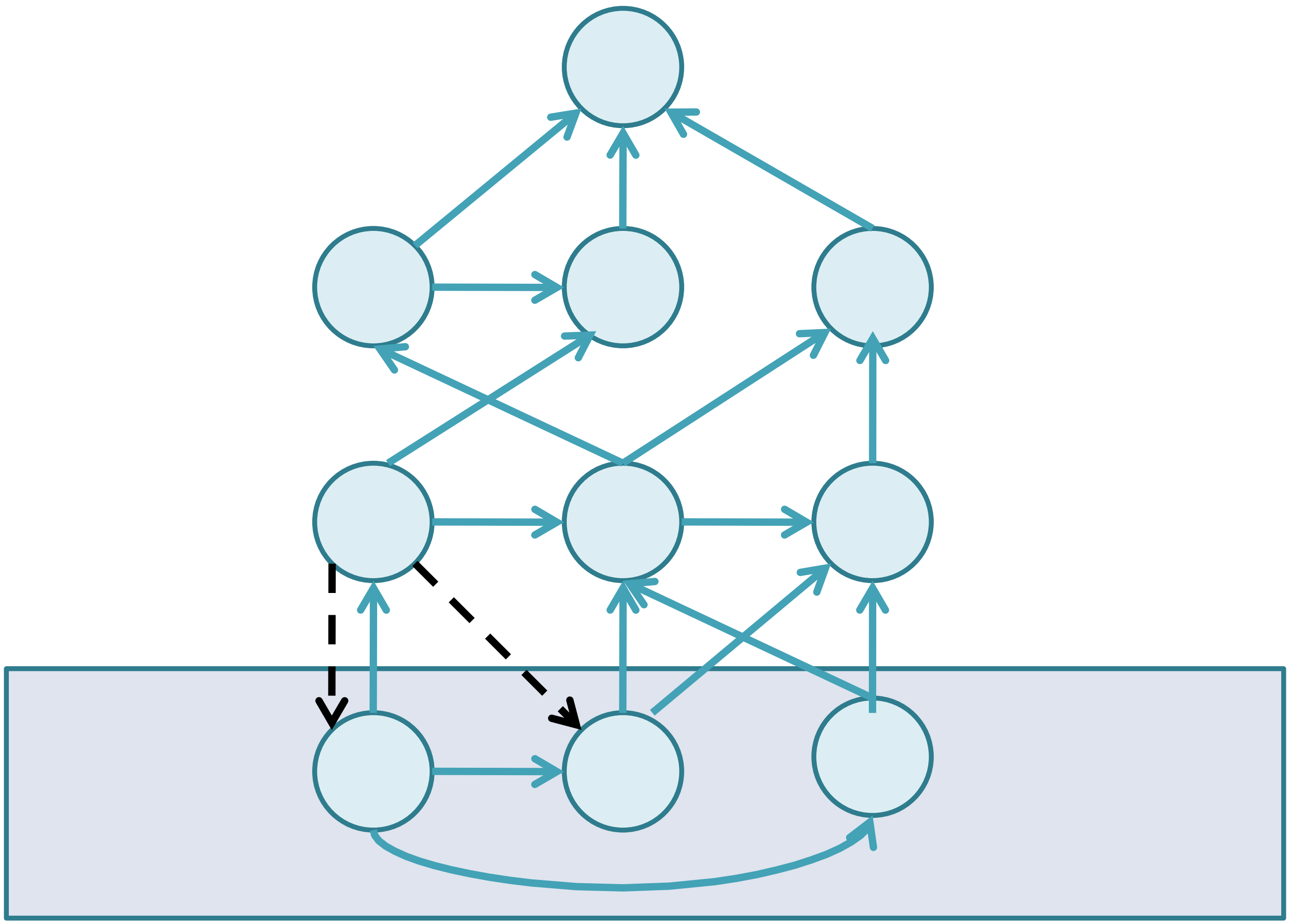


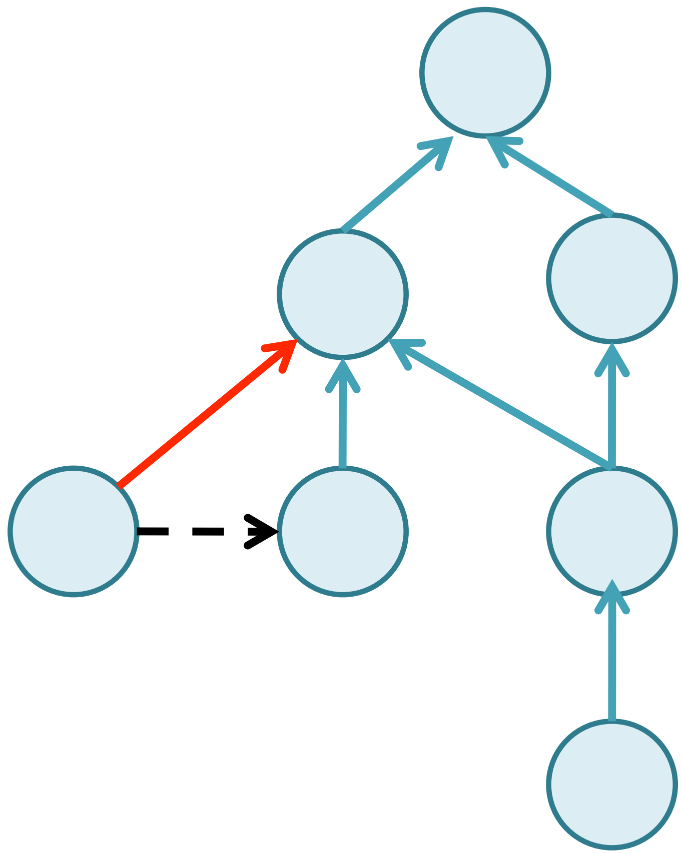
Find-Down-Edges

- For each node u at current level
  - Sort each node v_i in C (complete set) by distance to the root in G – {u}
  - Let v_1 … v_k be the sorted v_is
  - Let pi_1 … pi_k be their corresponding shortest paths to the root in G – {u}
  - For i from 1 to k
    - Do experiment of firing u, leaving pi_i free, and suppressing the rest of the nodes.

For Example



With Ones – a Problem



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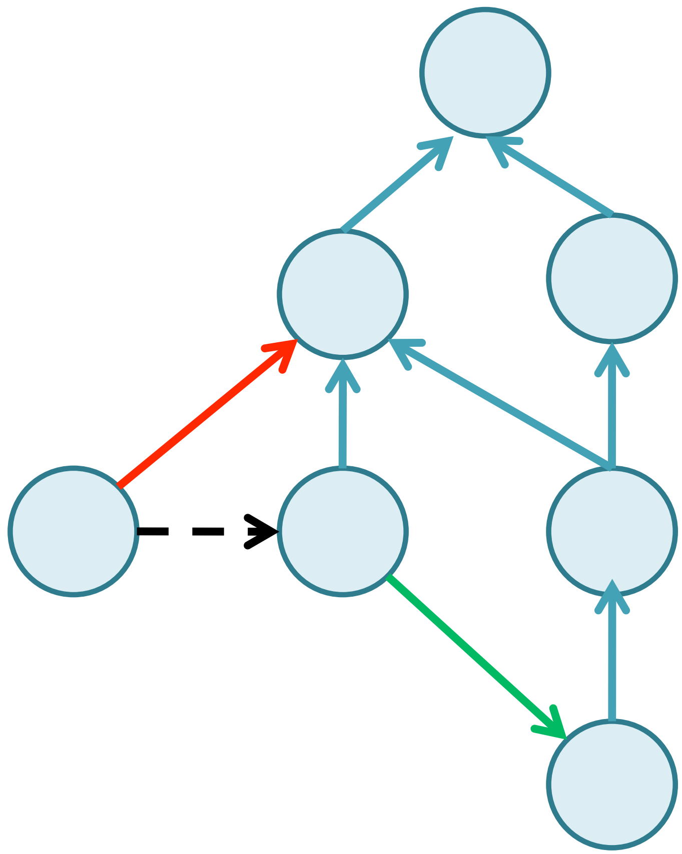


With Ones

- Algorithm gets more complicated
- Level edges and down edges are found in one subroutine
- In looking for down edges from u, need to avoid not just u, but also all nodes reachable from u by 1 edges

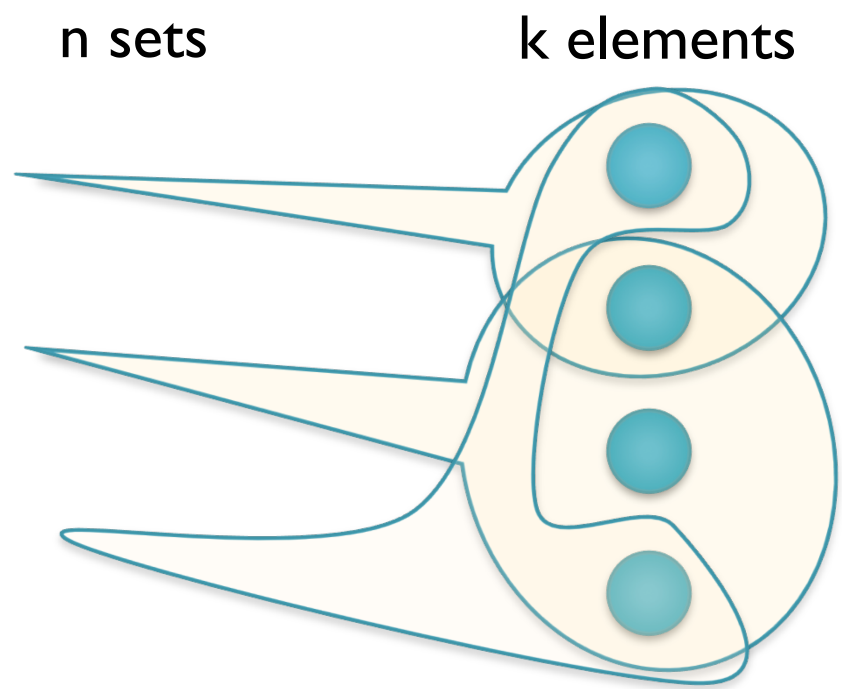
In the End

- We do 1 query per each possible edge, giving an $O(n^2)$ algorithm
- Matches the $\Omega(n^2)$ lower bound

Finding Influential Nodes

- Suppose instead of learning the social network, we wanted to find an influential set of nodes quickly.
- A set of nodes is influential if, when activated, activates the output with probability at least p
- NP Hard to Approximate to log n, even if we know the structure of the network

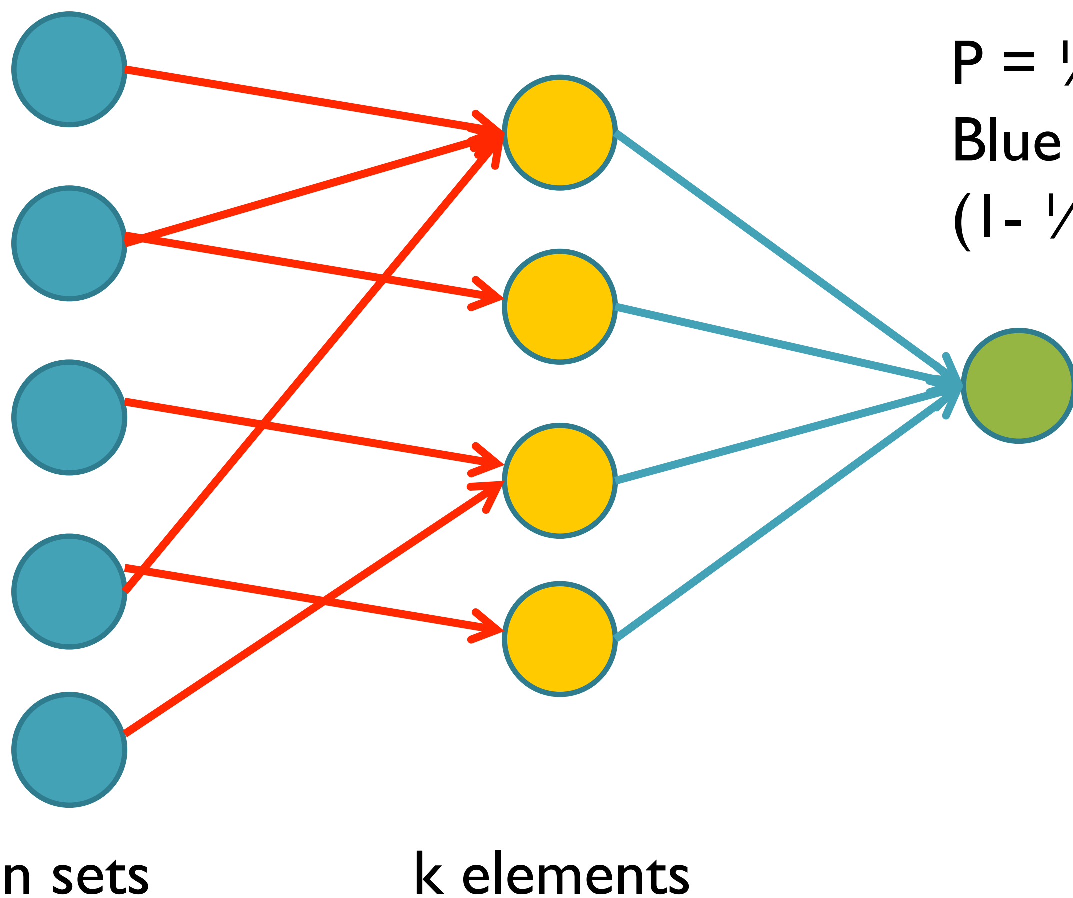
Set Cover



Can assume n = poly(k)

NP Hard to Approximate to log n

Reduction from Set Cover



P = ½
Blue edges have weight
(1- ½ ^(1/k))

An Approximation Algorithm

- Say the optimal solution has m nodes
- Suppose we wanted to fire the output with probability $(p - \varepsilon)$
- Let I be the set of chosen influential nodes.
- Observation: at any point in the algorithm, greedily adding one more node w to I makes

$$S(e_{I \cup \{w\}}) \geq S(e_I) + \frac{p - S(e_I)}{m}$$

Analyzing Greedy

- Using a greedy algorithm, we let k be the number of rounds the algorithm is run

For

$$p\left(1-\frac{1}{m}\right)^k < \epsilon$$

it suffices that

$$e^{-\frac{k}{m}} < \frac{\epsilon}{p}$$

or

$$k > m\log\left(\frac{p}{\epsilon}\right).$$

Summary of the Active Case

- Applies known model to new domain.
- Matching worst-case upper and lower bounds for learning social networks.
- But queries too expensive in most applications…
- Lots of open problems!

What if We Cannot Manipulate the Network?

2009 Cases of Swine Flu

The Constraints

- The social network is an unknown graph, where nodes are agents.
- Let $p_{(u,v)}$ be the a priori probability of an edge between nodes u and v.
- Each observed outbreak induces (or exposes) a constraint.
  - Namely the graph is connected on the induced subset.

Finding the Cheapest Network

- If the prior distribution is independent (and probabilities are small), the maximum likelihood social network maximizes

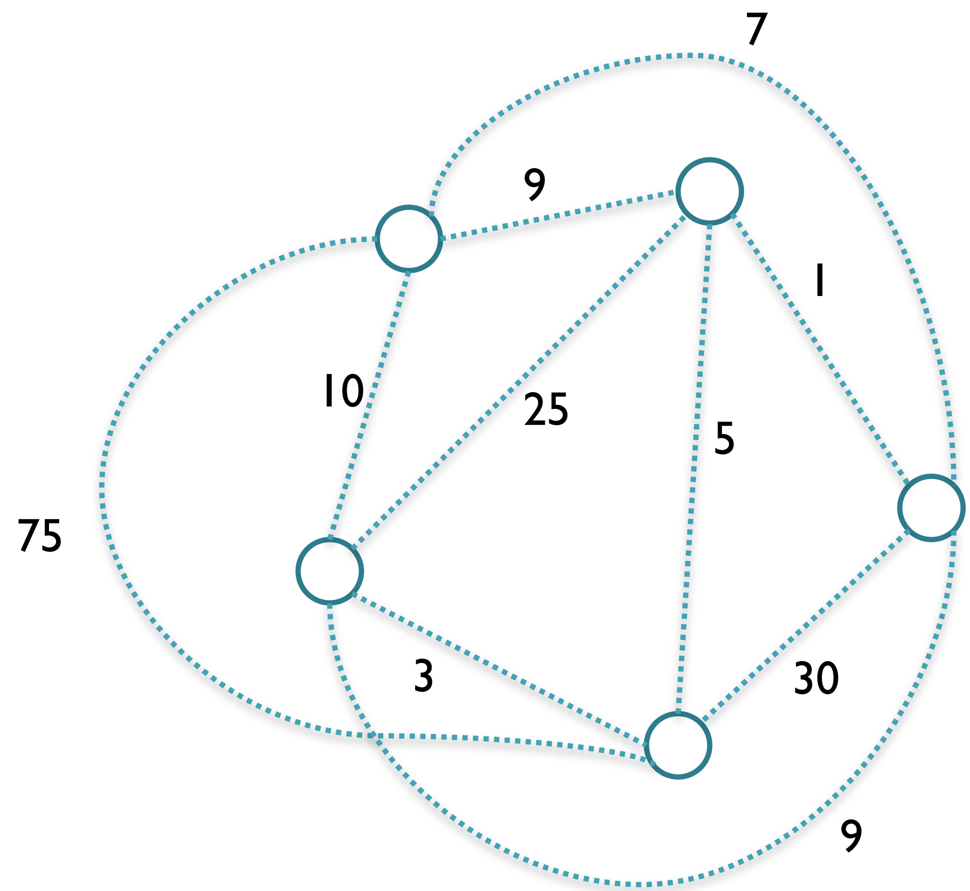
$$\prod_{u,v \in V} p_{(u,v)}$$

- This is equivalent to minimizing the sum of the log-likelihood costs

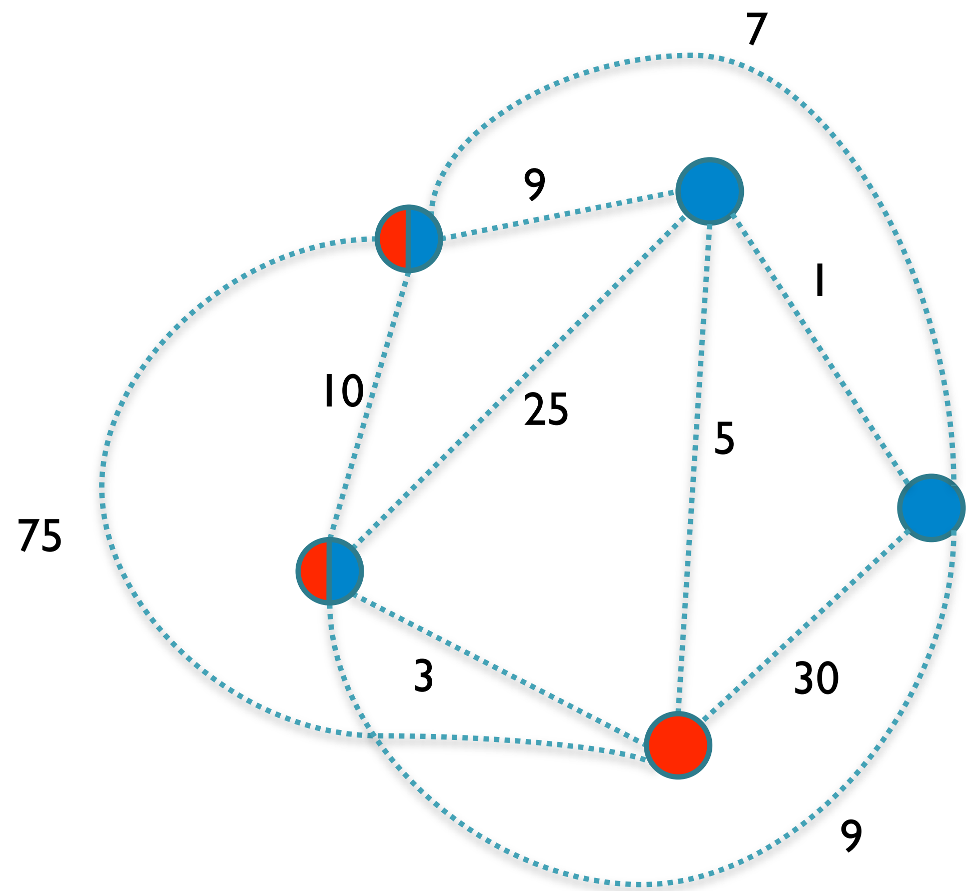
$$\sum_{v,u \in V} - \log(p_{(u,v)})$$

while satisfying the constraints

Finding the Cheapest Network Consistent with the Constraints



Finding the Cheapest Network Consistent with the Constraints



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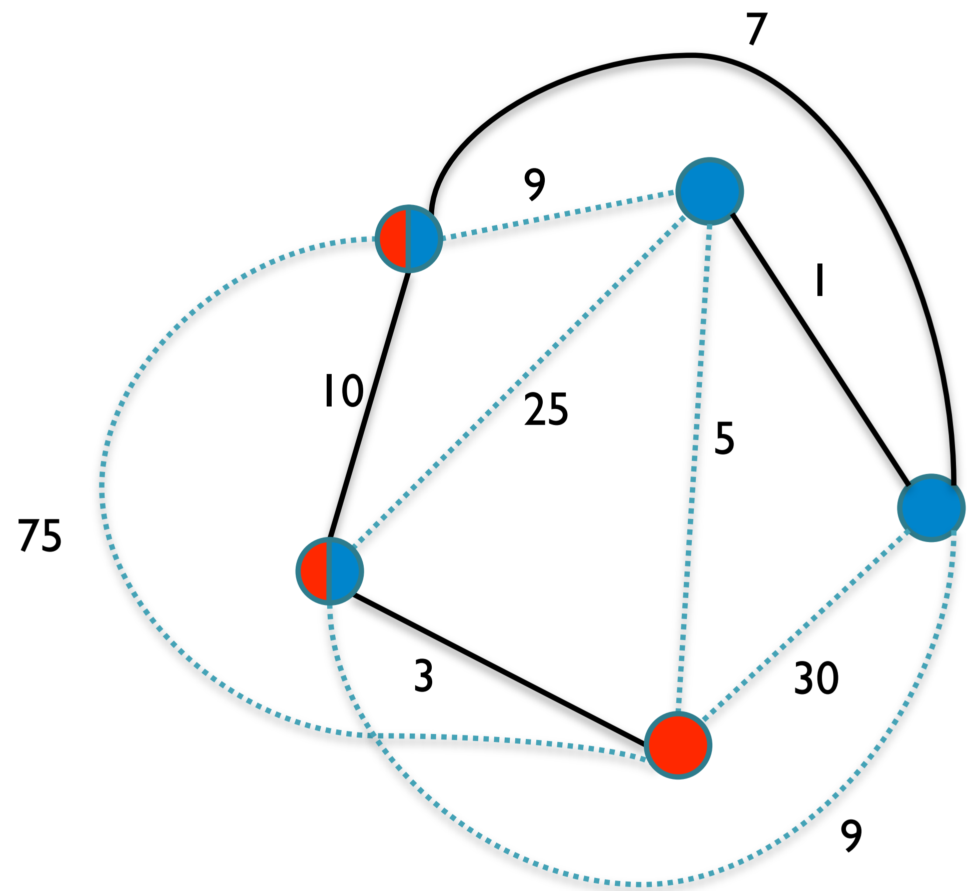

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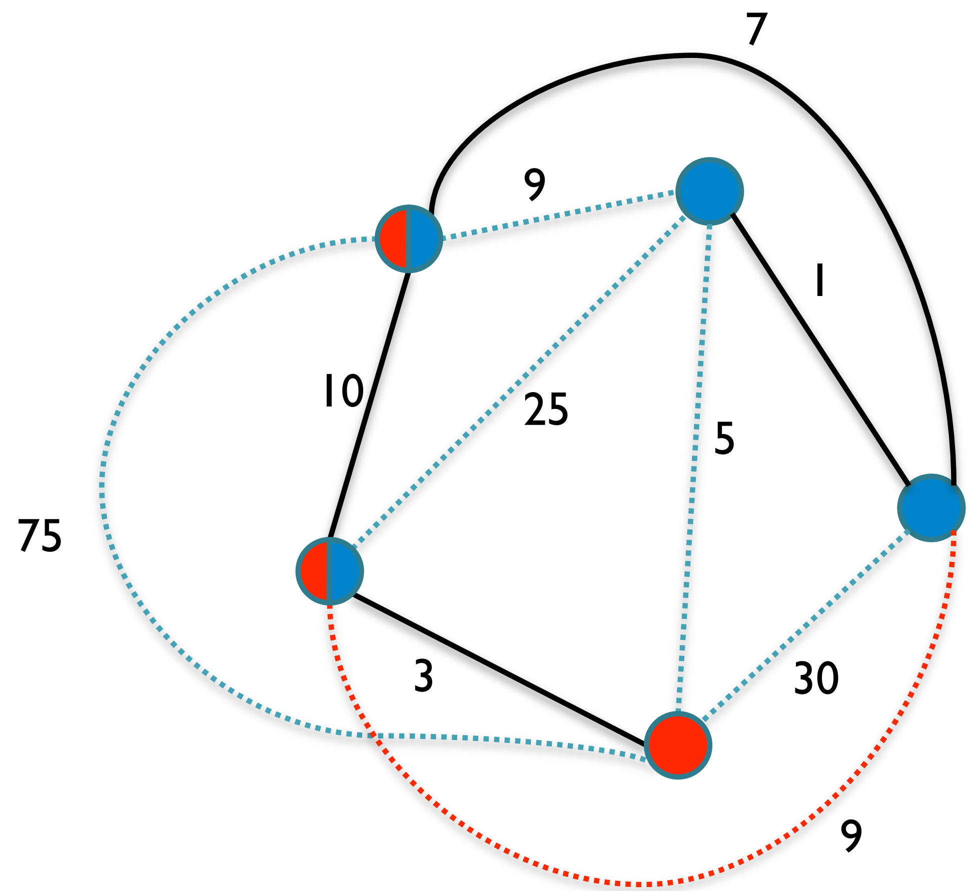


The Network Inference Problem

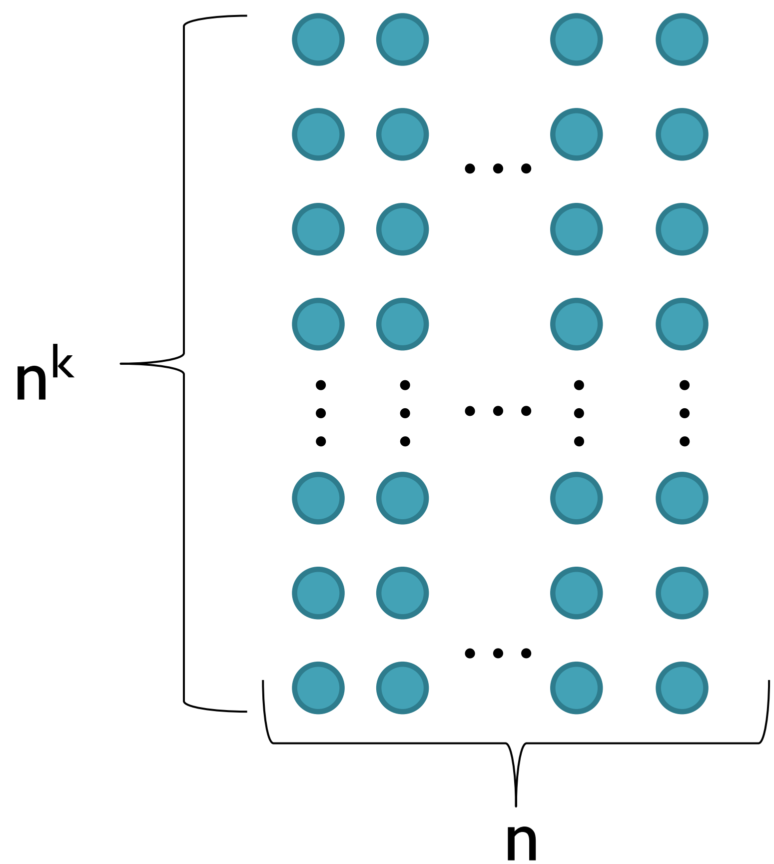
- The Network Inference Problem.
  - Given:
    - a set of vertices $V = \{v_1,\ldots,v_n\}$
    - costs c_e for each edge $e=\{v_i,v_j\}$
    - a constraint set $S = \{S_1,\ldots,S_r\}$, with $S_i \subseteq V$
  - Find: a set E of edges of lowest cost such that each S_i induces a connected subgraph of $G=(V,E)$
- We consider both the offline and online version of this problem. We also consider the arbitrary and uniform cost versions.
- Solved for the case where all constraints can be satisfied by a tree [Korach & Stern '03] – they left the general case open

An Offline Lower Bound

- Theorem: If P ≠ NP, the approximation ratio for the Uniform Cost Network Inference problem is $\Omega(\log n)$.
- Proof (reduction from Hitting Set)
  - $U = \{v_1, v_2, \ldots, v_n\}$
  - $C = \{C_1, C_2, \ldots, C_j\}$, with $C_i \subseteq U$
  - The Hitting Set problem is to minimize $|H|$, where $H \subseteq U \text{ s.t. } \forall C_i\ H \cap C_i \neq \phi$

An Offline L.B. continued

- Reduction from Hitting Set
- For a constant k, We make a N.I. instance



An Offline L.B. continued

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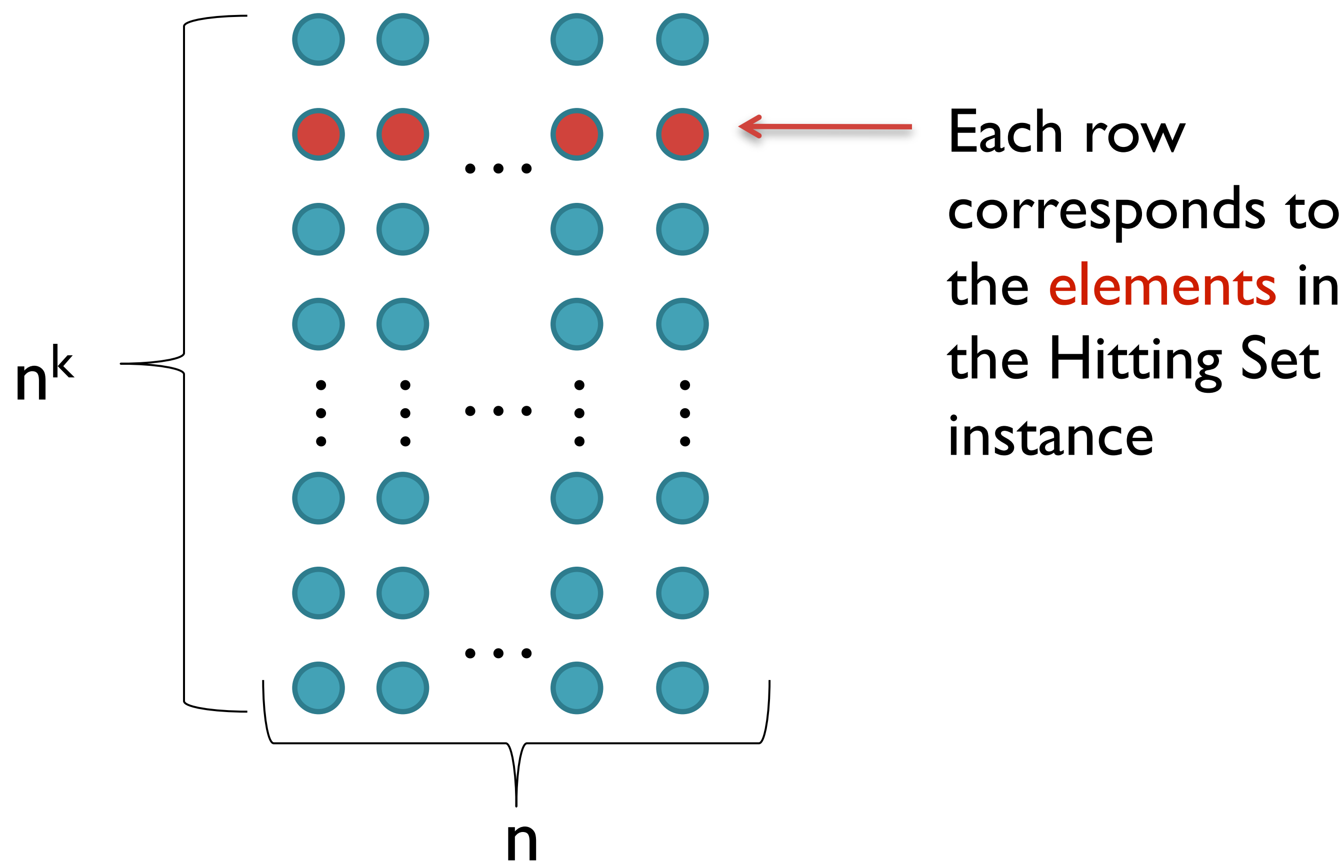
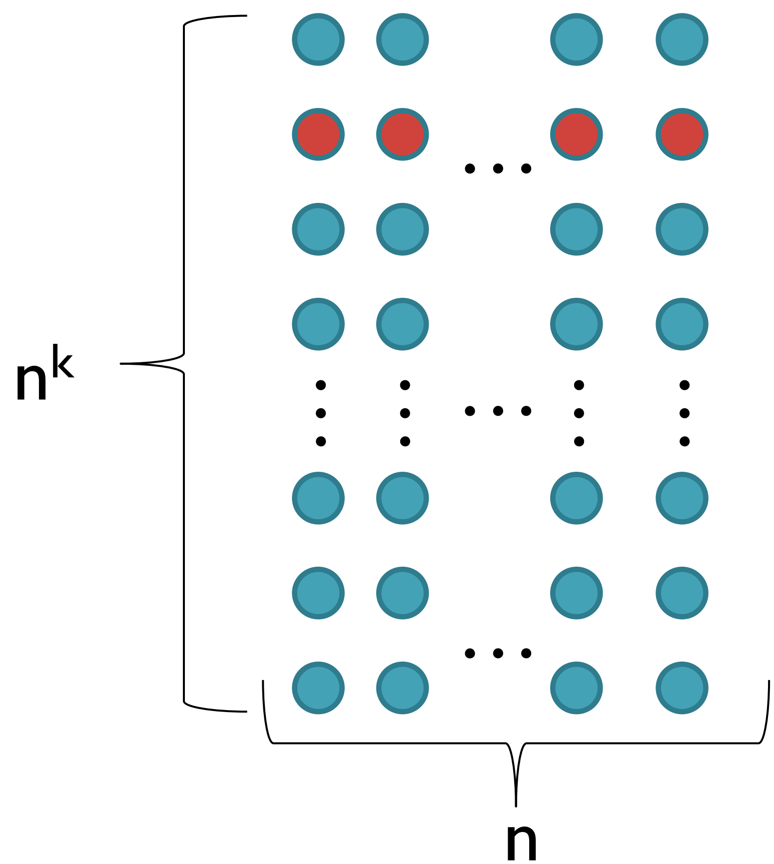


An Offline L.B. continued

- Constraints: first, for each row, give all pairwise constraints:



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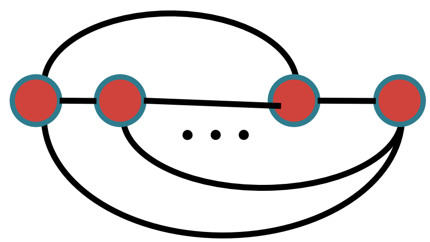
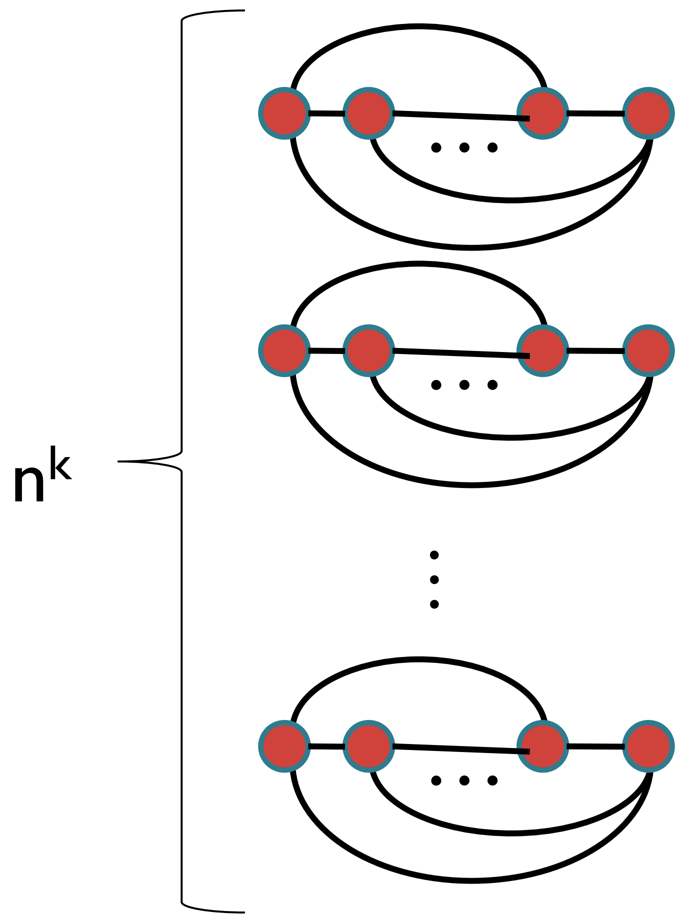


- This will force the learner to put down a clique on each row

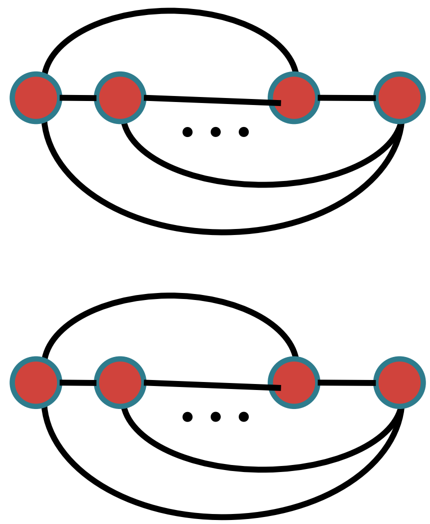
An Offline L.B. continued

- Now we have n^k rows of cliques



An Offline L.B. continued

- For each pair of rows:



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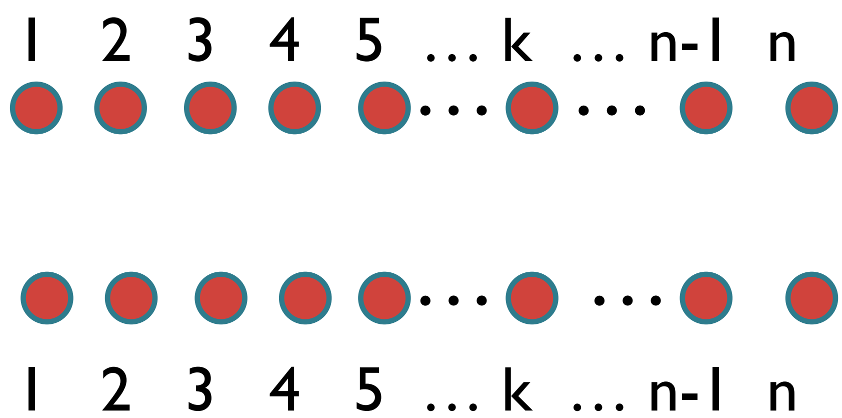

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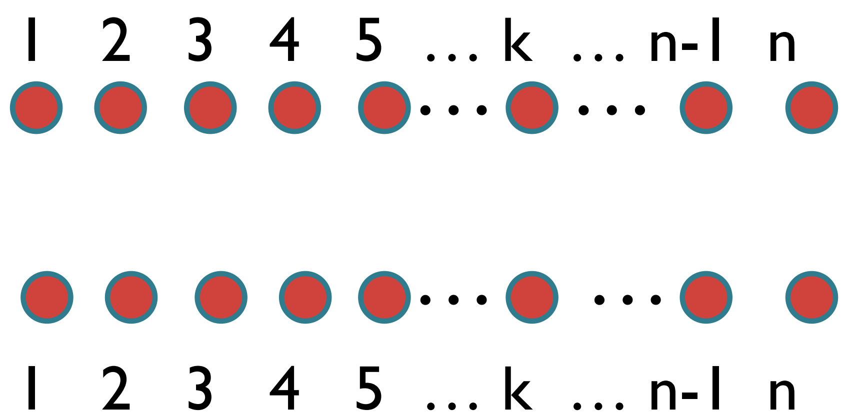
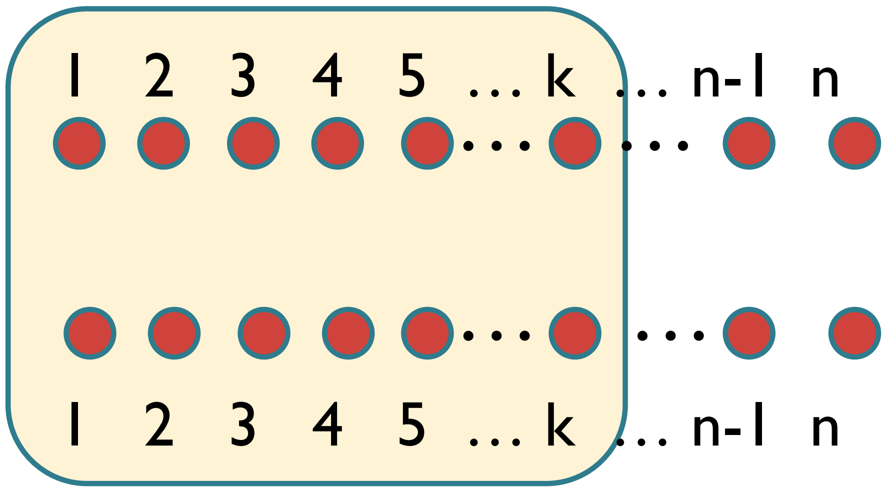


- w.l.o.g. for the Hitting Set constraint
  - $C_i = \{v_1, v_2, \ldots, v_k\}$
  - we will add the constraint:

An Offline L.B. continued

- For each pair of rows:



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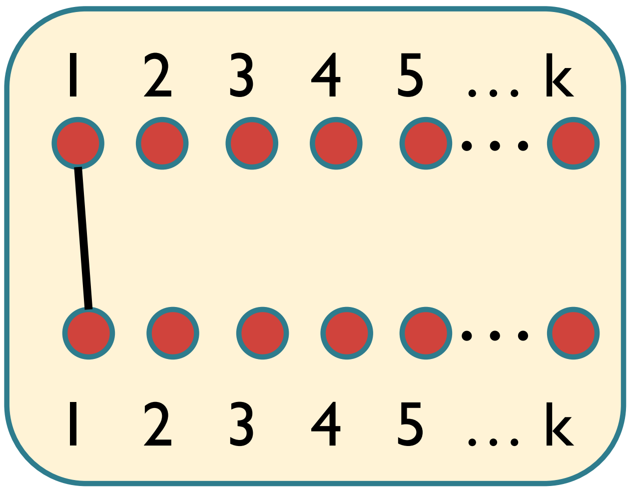
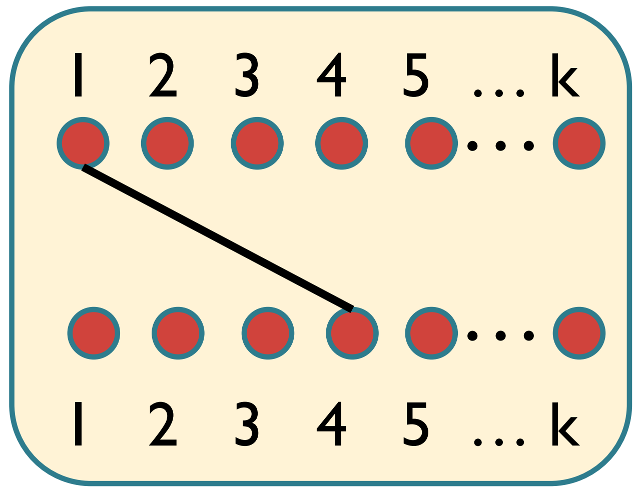
An Offline L.B. continued

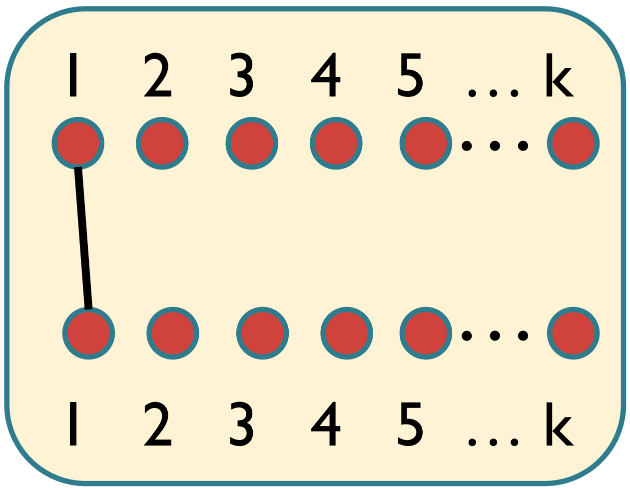


corresponds to adding v_1 to H



never better

An Offline L.B. continued



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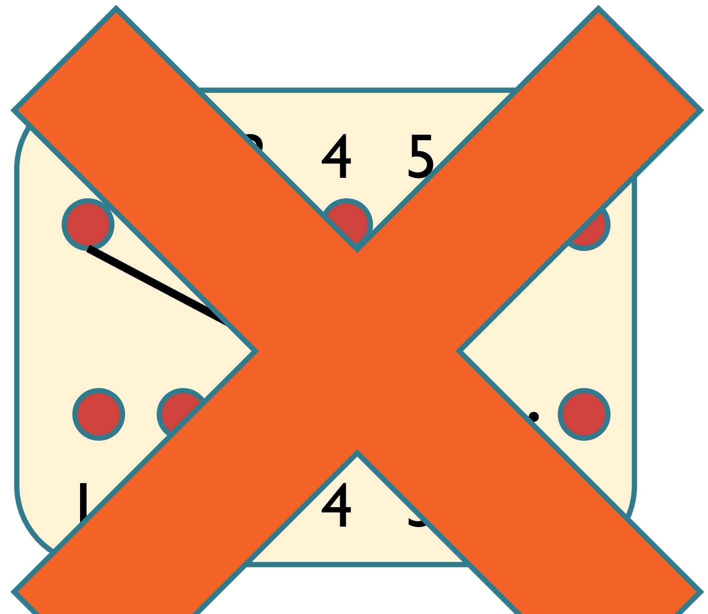


never better

Finishing the Lower Bound

- Unless P=NP, optimal Hitting Set approximation is Ω(log(n)) [Feige '98].
- The optimal algorithm pays:

$$n^k \binom{n}{2} + \mathrm{OPT}\binom{n^k}{2}$$

- But the learner pays:

$$n^k \binom{n}{2} + \Omega\left(\log(n)\mathrm{OPT}\binom{n^k}{2}\right)$$

- k can be chosen to be arbitrarily large.

Offline Network Inference Algorithm

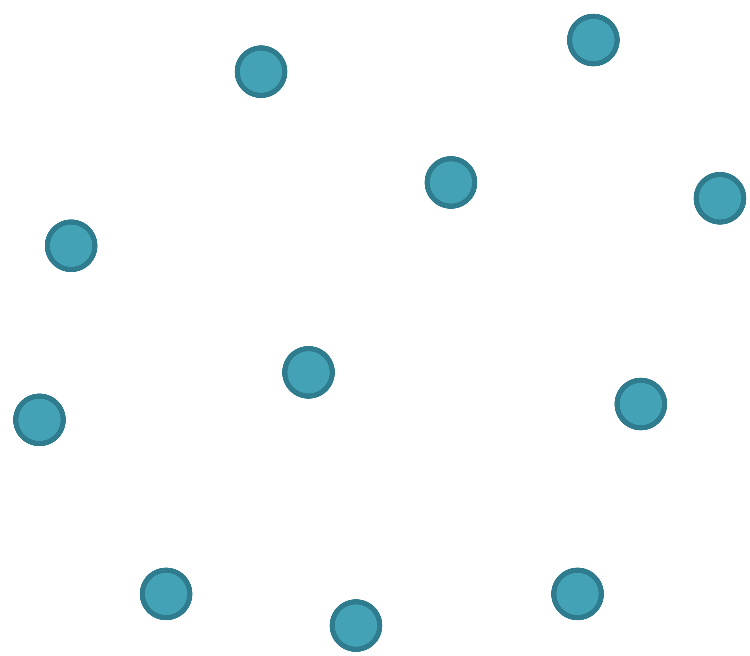
- Theorem: There is a $O(\log(n)+\log(r))$ approximation algorithm to OPT
- Proof:
  - Let **C** sum over all constraints S_i, the number of components S_i induces in G minus 1.
  - Now consider the greedy algorithm: while $C > 0$, add to E the edge that has the lowest ratio of c_e to ΔC.
  - This greedy algorithm gives an approximation of $\log(C_0) = O(\log(n)+\log(r))$

The Online Problem

- Constraints S_i come in online
- Must satisfy each constraint as it comes in.
- Can add but not remove edges.
- Seemingly good ideas like placing a spanning tree on each constraint can perform very badly.

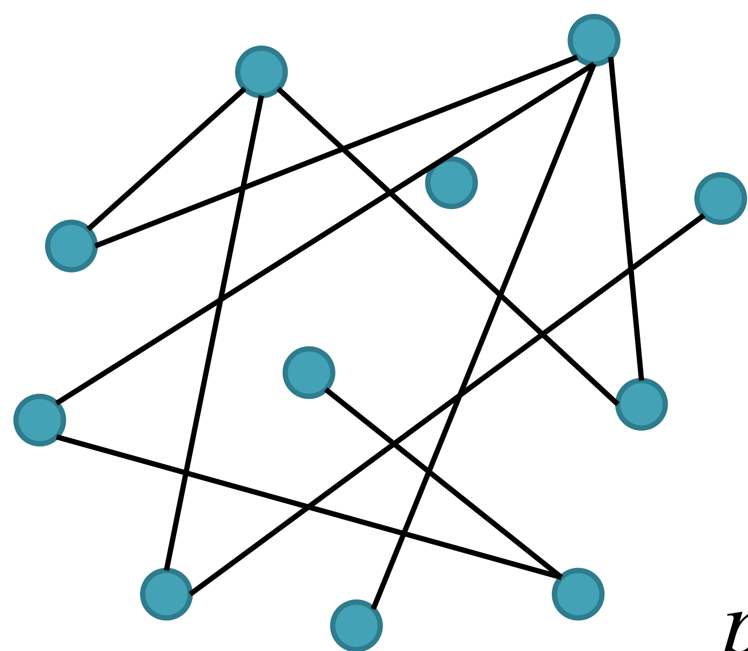
Online Algorithm Against Oblivious Adversary

$O(n^{2/3}\log^{2/3}n)$-competitive algorithm



Online Algorithm Against Oblivious Adversary

$O(n^{2/3}\log^{2/3} n)$-competitive algorithm



$$p = \frac{c\log^{2/3} n}{n^{1/3}}$$

Online Algorithm Against Oblivious Adversary

$O(n^{2/3}\log^{2/3}n)$-competitive algorithm

- All constraints S_i, $|S_i| \geq n^{1/3}\log^{1/3}(n)$ are almost surely connected
- All constraints S_i, $|S_i| < n^{1/3}\log^{1/3}(n)$ that are not already covered, we can put a clique on, and hit at least 1 edge in OPT
- We used $O(n^{5/3}\log^{2/3}(n)+n^{2/3}\log^{2/3}(n)\mathrm{OPT})$ edges in expectation.
- Because OPT = $\Omega(n)$, we are done.

Other Online Results

- The competitive ratio for uniform cost stars and paths is θ(log n).
  - for paths, makes use of pq-trees [Booth and Lueker '76]

- The uniform cost problem has a $\Omega\left(\sqrt{n}\right)$-competitive lower bound

- The arbitrary cost problem has an Ω(n)-competitive lower bound and O(n log n)-competitive algorithm.

Summary

- Lots of other results in this model.
- Passive model does not require interfering in the network.
- Interesting techniques, but gaps left.
- Would be interesting to extend to models incorporating incomplete observations.
- Extend to weaker adversaries or random networks.

Thank You!

Questions?