Learning Circuits and Networks by Injecting Values

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Papers

- Dana Angluin, James Aspnes, Jiang Chen, and Yinghua Wu. **Learning a Circuit by Injecting Values.** In *Proceedings of the 38th ACM Symposium on Theory of Computing* (STOC), May 2006

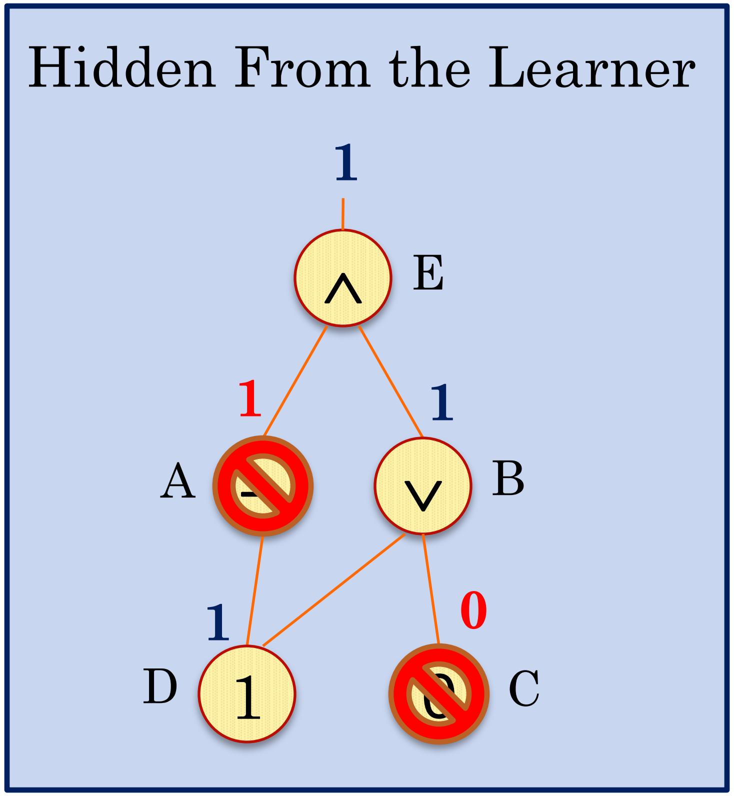
- Dana Angluin, James Aspnes, Jiang Chen, and Lev Reyzin. **Learning Large-Alphabet and Analog Circuits with Value Injection Queries.** In *Proceedings of the 20th Annual Conference on Learning Theory* (COLT), June 2007

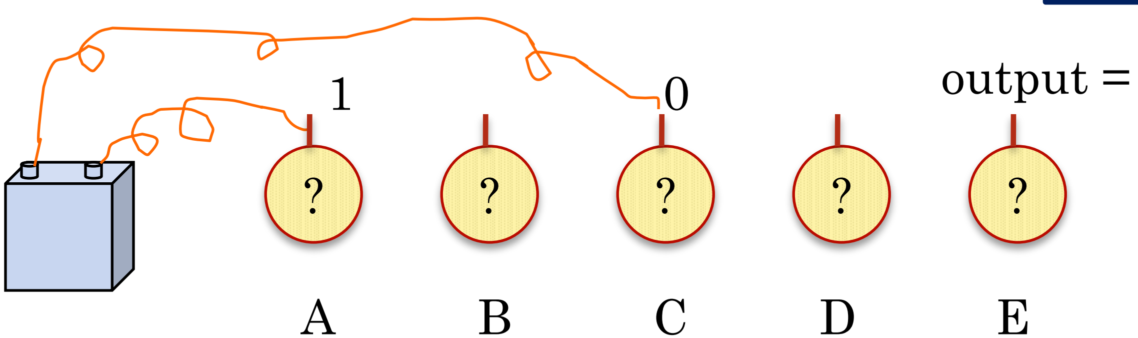
- Dana Angluin, James Aspnes, Jiang Chen, David Eisenstat, and Lev Reyzin. **Learning Acyclic Probabilistic Circuits Using Test Paths.** To appear in *Proceedings of the 21st Annual Conference on Learning Theory* (COLT), July 2008

- Dana Angluin, James Aspnes, and Lev Reyzin. **Optimally Learning Social Networks with Activations and Suppressions.** *In preparation*, February 2008

The Value Injection Query Model

- Introduced by [AACW '06]
- Experiments on a hidden Circuit.
  - a gate output may be fixed
  - a gate may be left free
- Query
  - given an experiment, we can observe its output
- Example:

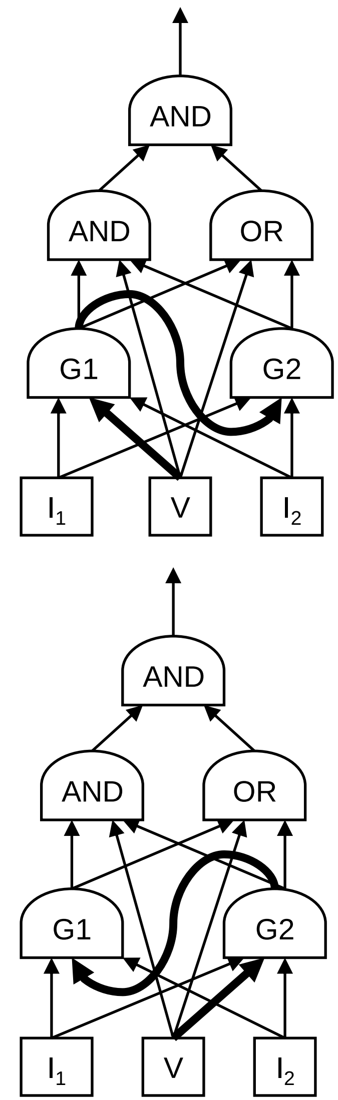




THE LEARNING PROBLEM

- **Behavioral equivalence:** Two circuits C and C' are behaviorally equivalent if for any experiment s, C(s)=C'(s).

- **The Problem:** Given qeuery access to a hidden circuit C*, find a circuit C behaviorally equivalent to C* by making value-injection queries.



[ACCW '06]

Motivation for The Model

- To model gene regulatory networks as boolean networks
- to represent gene expressions and disruptions

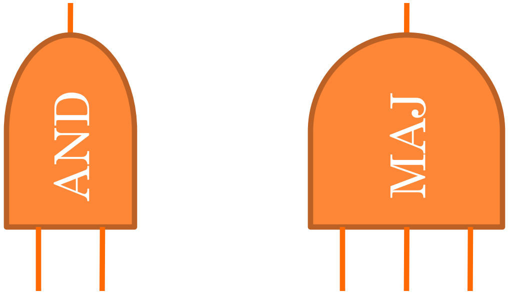
Previous gene regulatory network model	Fully controllable.	All gates are observable.
Existing circuit learning models	Only inputs can be manipulated.	Only the output is observable.
[AACW ’06] model	Fully controllable.	Only the output is observable.

[AACW '06] Results for Boolean Circuits

Depth	**Fan-in**	**Gates**	**Learnability**
Unbounded	Unbounded	AND/OR	$2^{\Omega(N)}$ queries
Unbounded	2	AND/OR	NP-hard
Constant	Unbounded	AND/OR/Θ_2	NP-hard
Log	Constant	Arbitrary	Poly-time **(NC1)**
Constant	Unbounded	AND/OR/NOT	Poly-time **(AC0)**

Large-Alphabet Circuits

Gates in Boolean Circuits



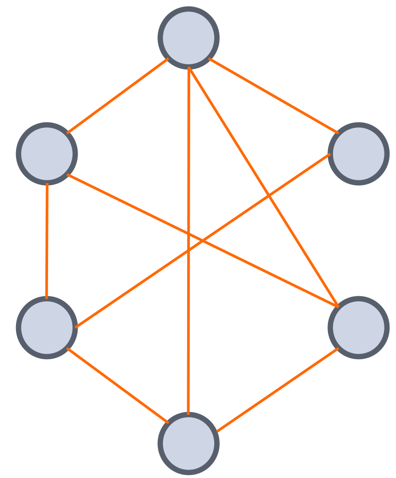
Input 1	Input 2	Output
1	1	O_1
1	0	O_2
0	1	O_3
0	0	O_4

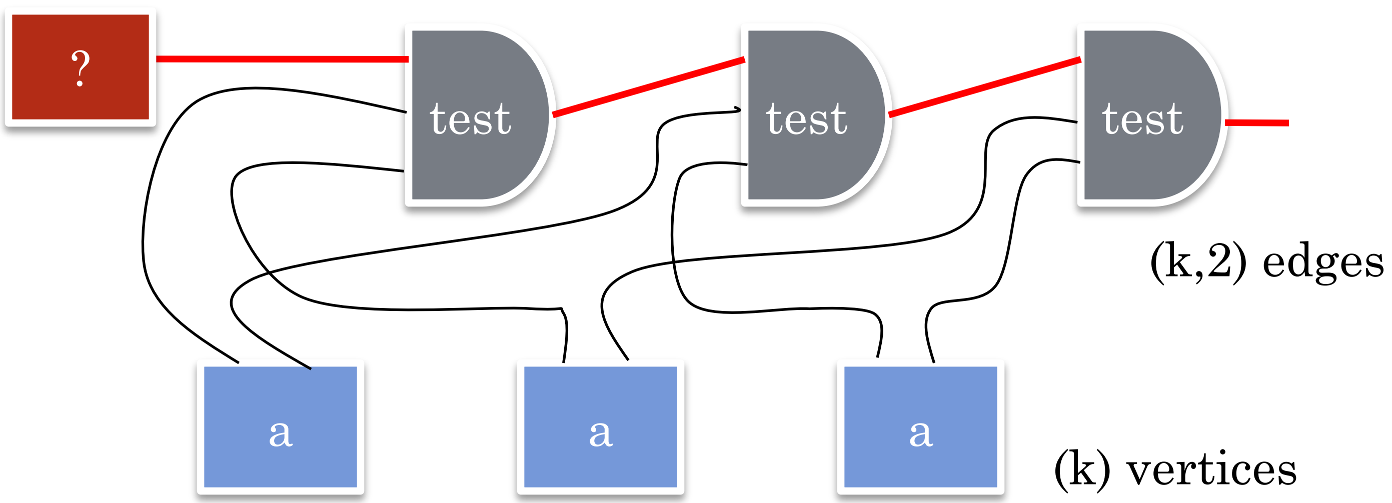
Gates in Large-Alphabet circuits

Input 1	Input 2	Output
A	A	O_1
A	B	O_2
A	C	O_3
B	A	O_4
B	B	O_5
B	C	O_6
C	A	O_7
C	B	O_8
C	C	O_9

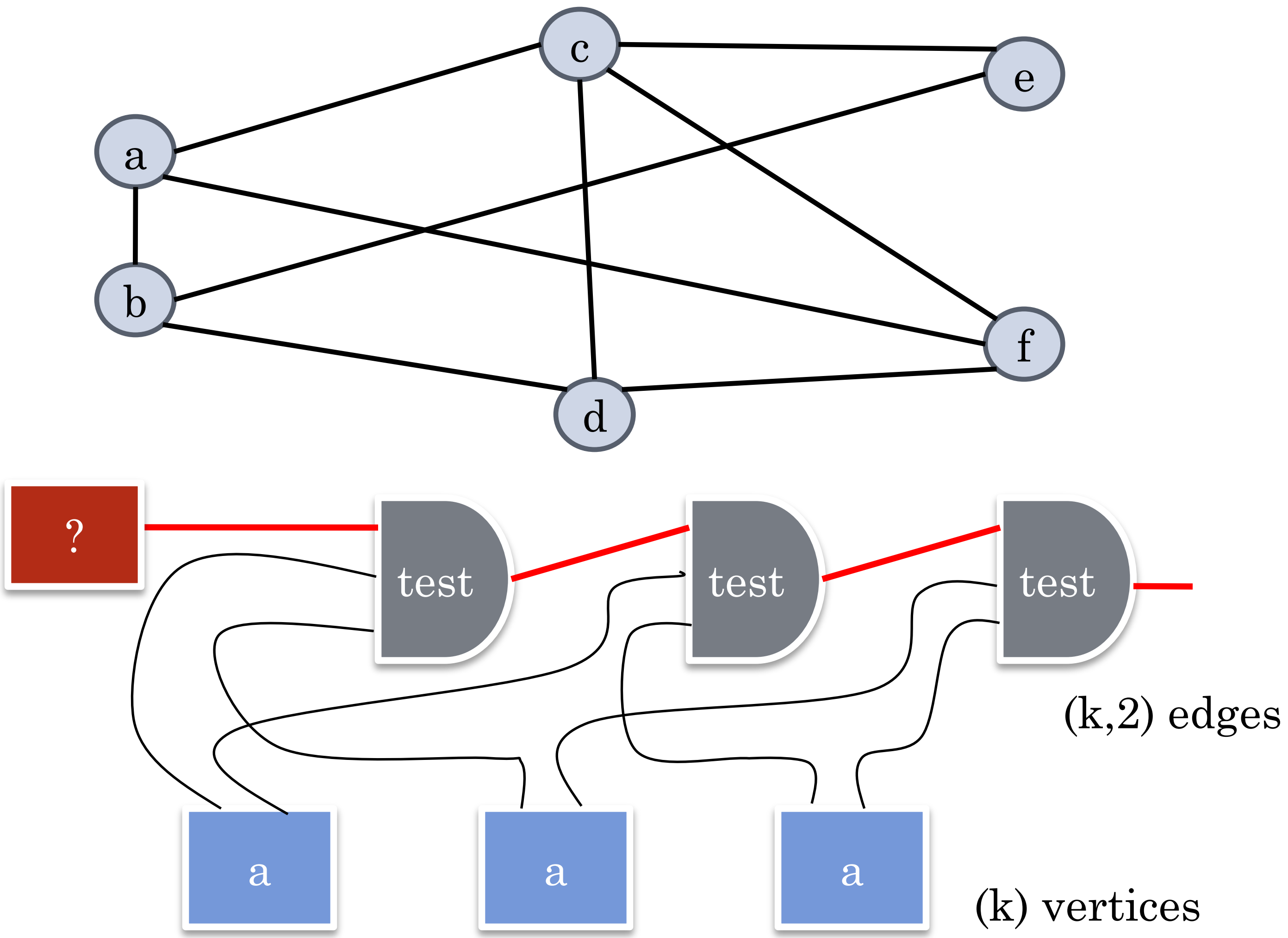
Hardness of Learning Large Alphabet Circuits

- Consider the problem on input (G,k) of telling whether the graph G on n vertices has a clique of size k
- We give a reduction that turns a large-alphabet circuit learning algorithm into a clique tester





Reducing the Clique Problem to Circuit Learning



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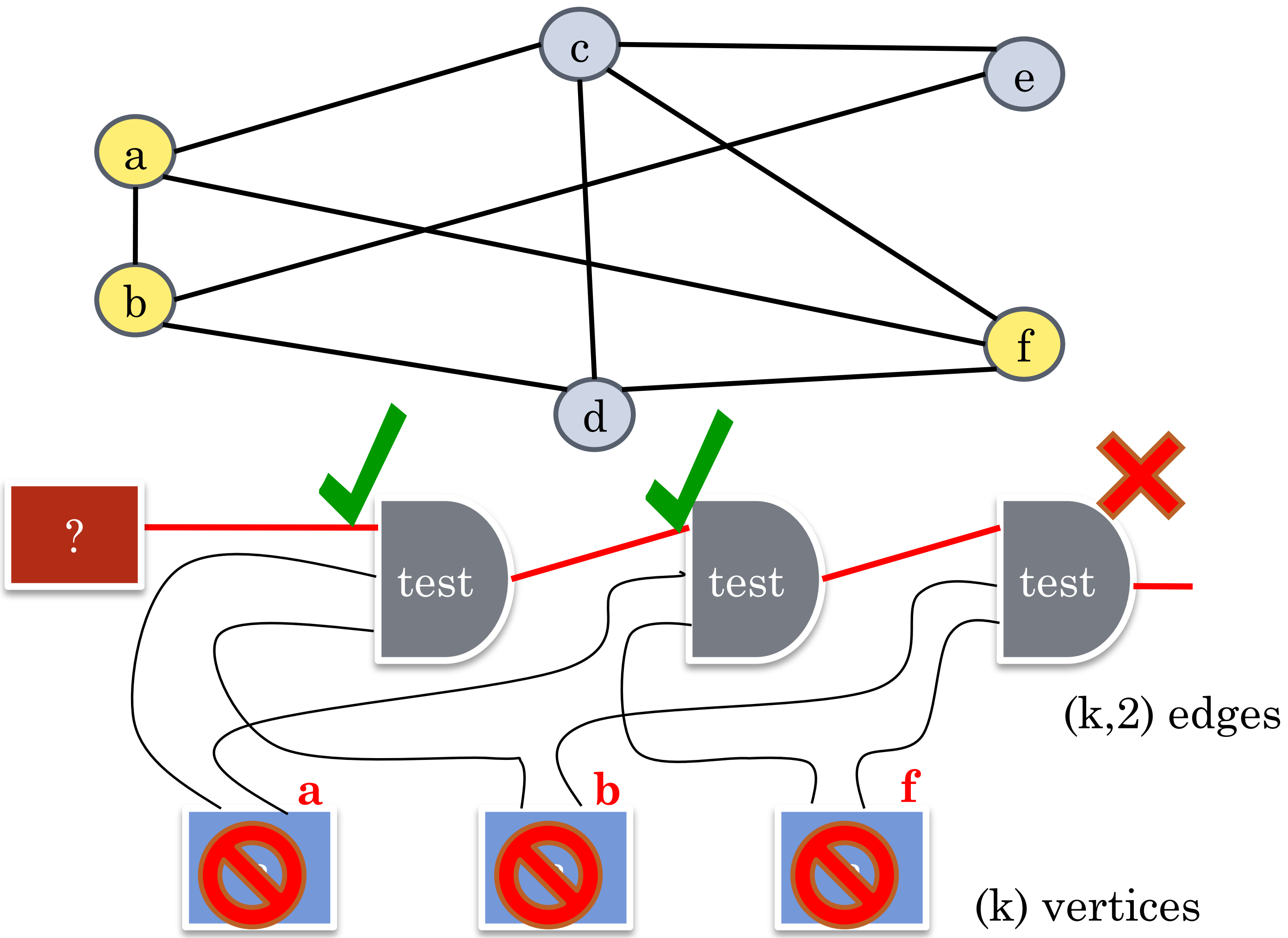

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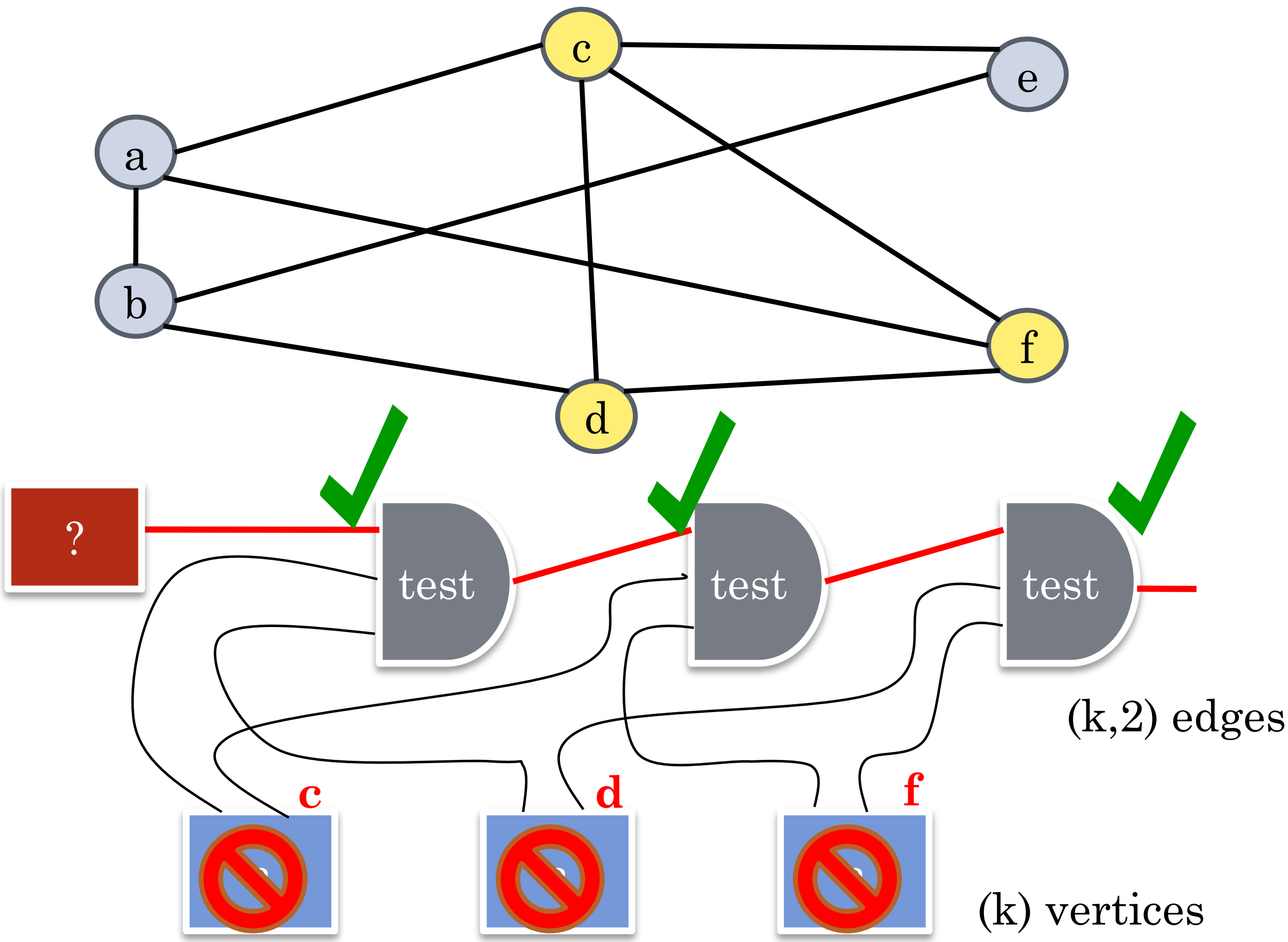


Hardness of Learning Circuits of Unrestricted Topology

- The clique problem is complete for the **parameterized complexity class** W[1]
  - There is no known algorithm for the clique problem that runs in time $f(k)n^c$ (and we believe one doesn't exist)

- **Theorem** **An algorithm for learning circuits polynomial in the number of wires and alphabet size would imply fixed parameter tractability for all problems in W[1]**

To Compare with the Boolean Case

Boolean Circuits [AACW '06]:

Depth	Fan-in	Gates	Learnability
Log	Constant	Arbitrary	Poly-time

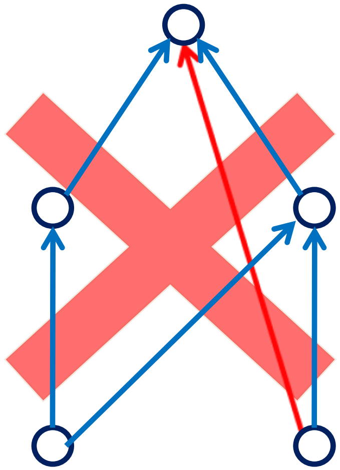
Large Alphabet Circuits:

Depth	Fan-in	Gates	Learnability
Log	Constant	Arbitrary	W[1] Hard

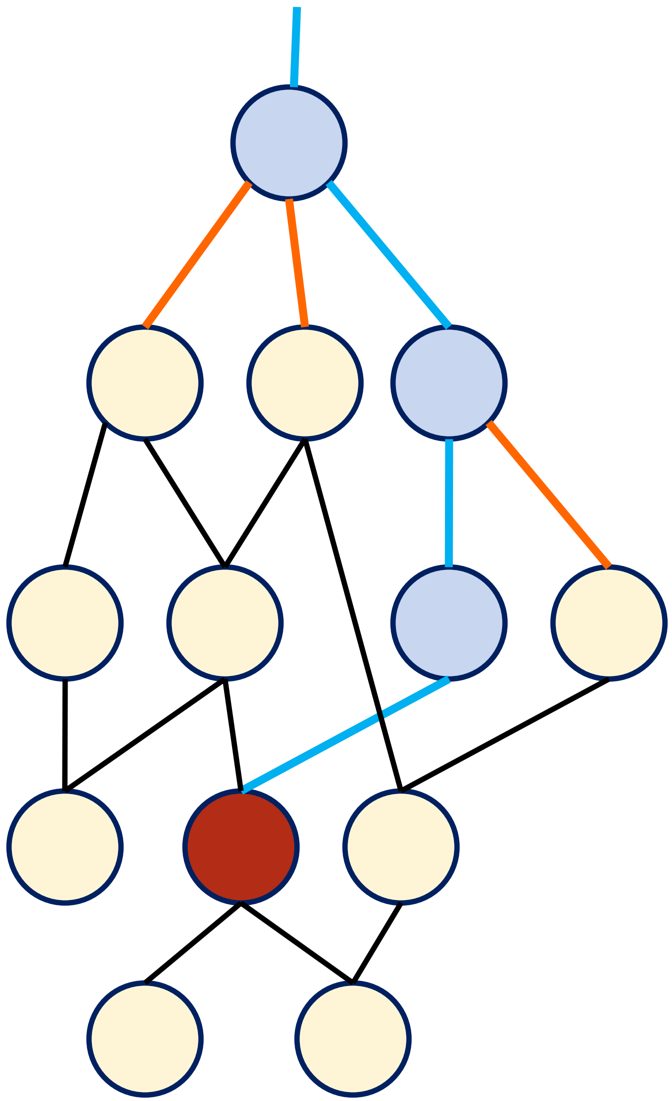
This motivates looking at classes of large-alphabet circuits with restricted topology

TRANSITIVELY REDUCED CIRCUITS

A circuit is transitively reduced if its underlying directed graph has no shortcuts. If (u,v) is an edge and there is a path of length ≥ 2 from u to v, then (u,v) is a **shortcut edge**



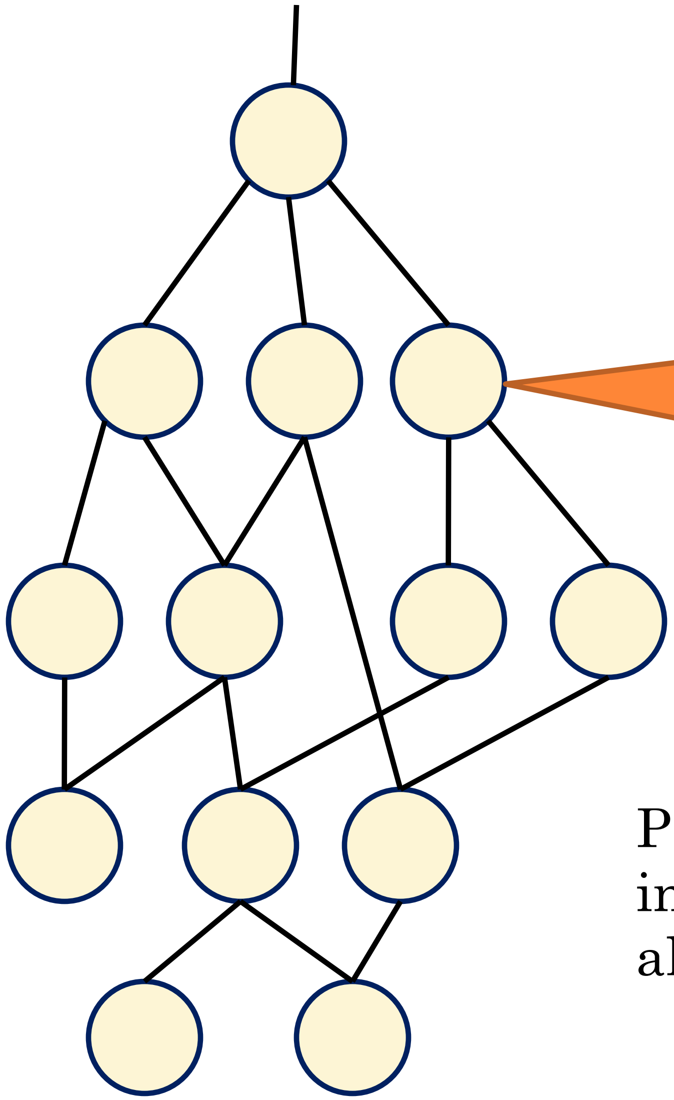
Paths to the Root



Large Alphabet Circuit Result

Theorem **We can learn the class of circuits having n wires, alphabet size s, fan-in bound k, and shortcut width bounded by b, using $ns^{O(k+b)}$ value injection queries and time polynomial in the number of queries.**

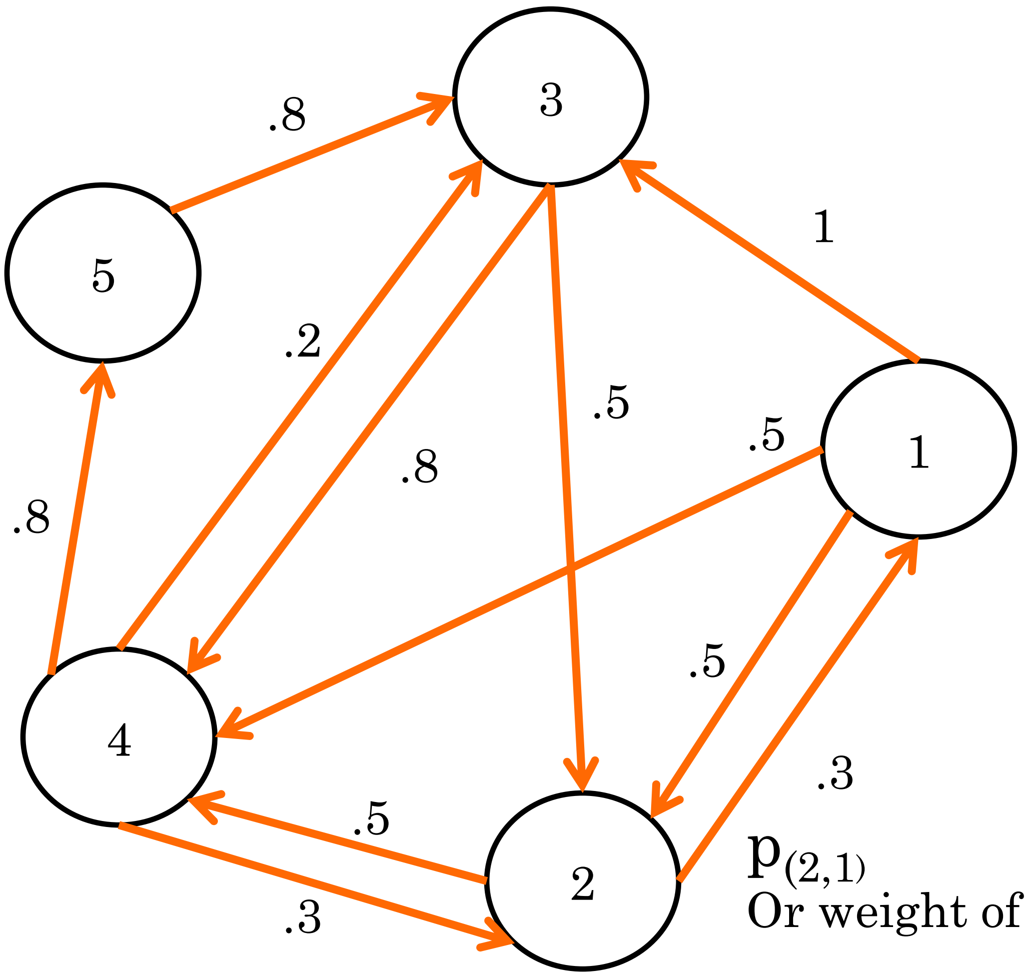
PROBABILISTIC CIRCUITS



	P(0)	P(1)
00	.5	.5
01	1	0
10	.8	.2
11	.3	.7

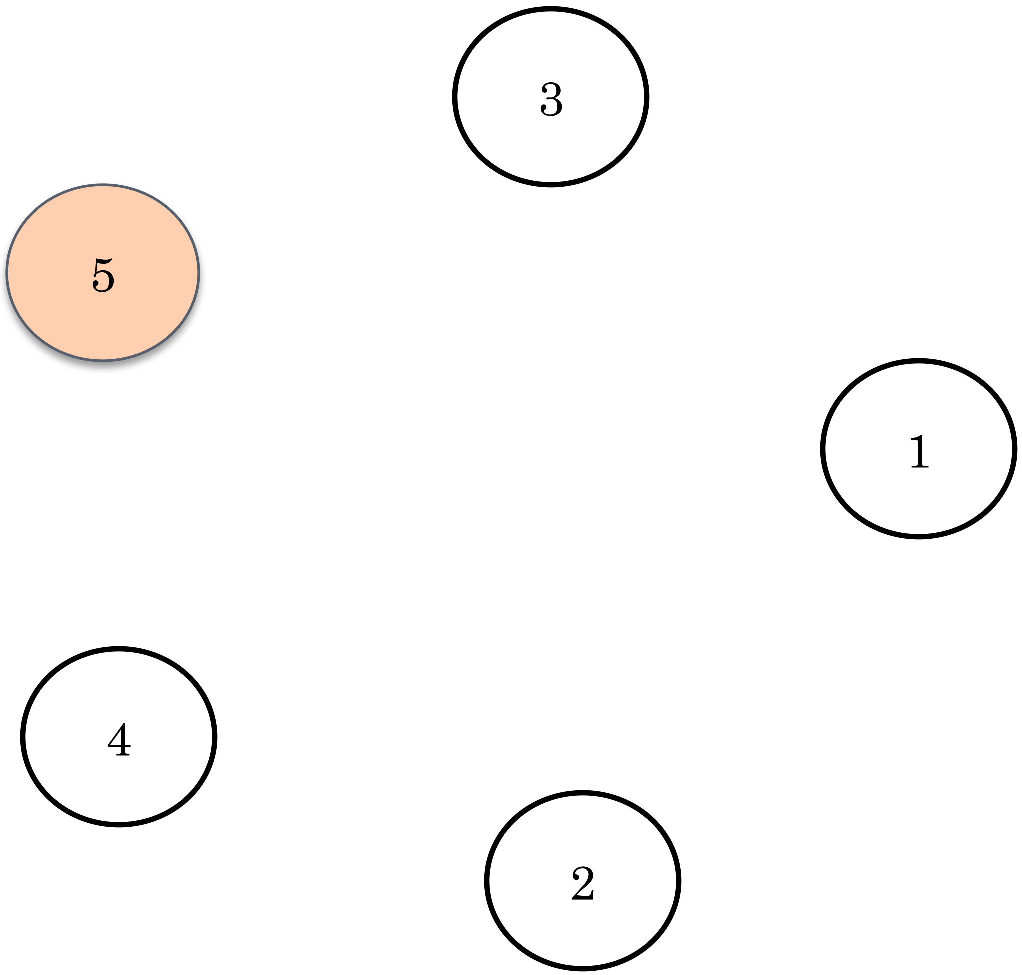
Path-based methods no longer work in the probabilistic case (for large alphabets).

Independent Cascade Social Networks

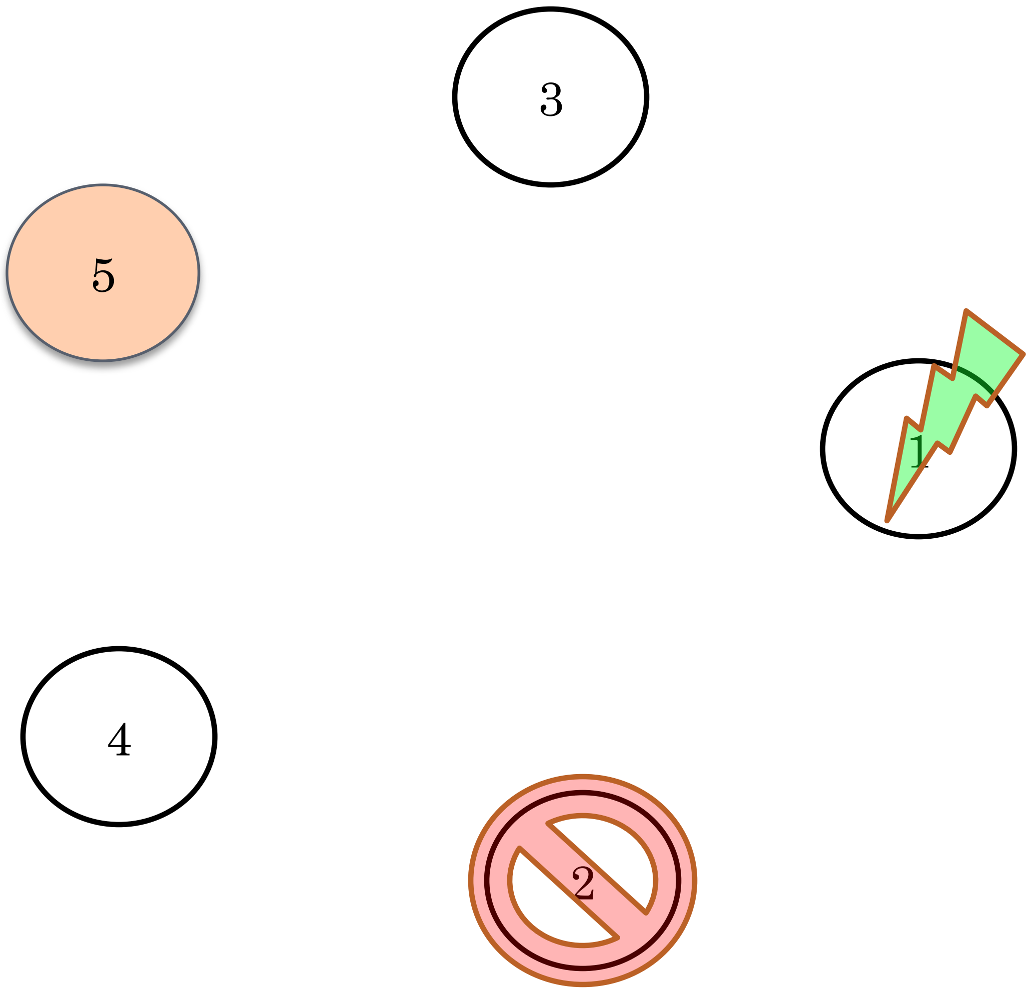


$p_{(2,1)}$
Or weight of edge (2,1)

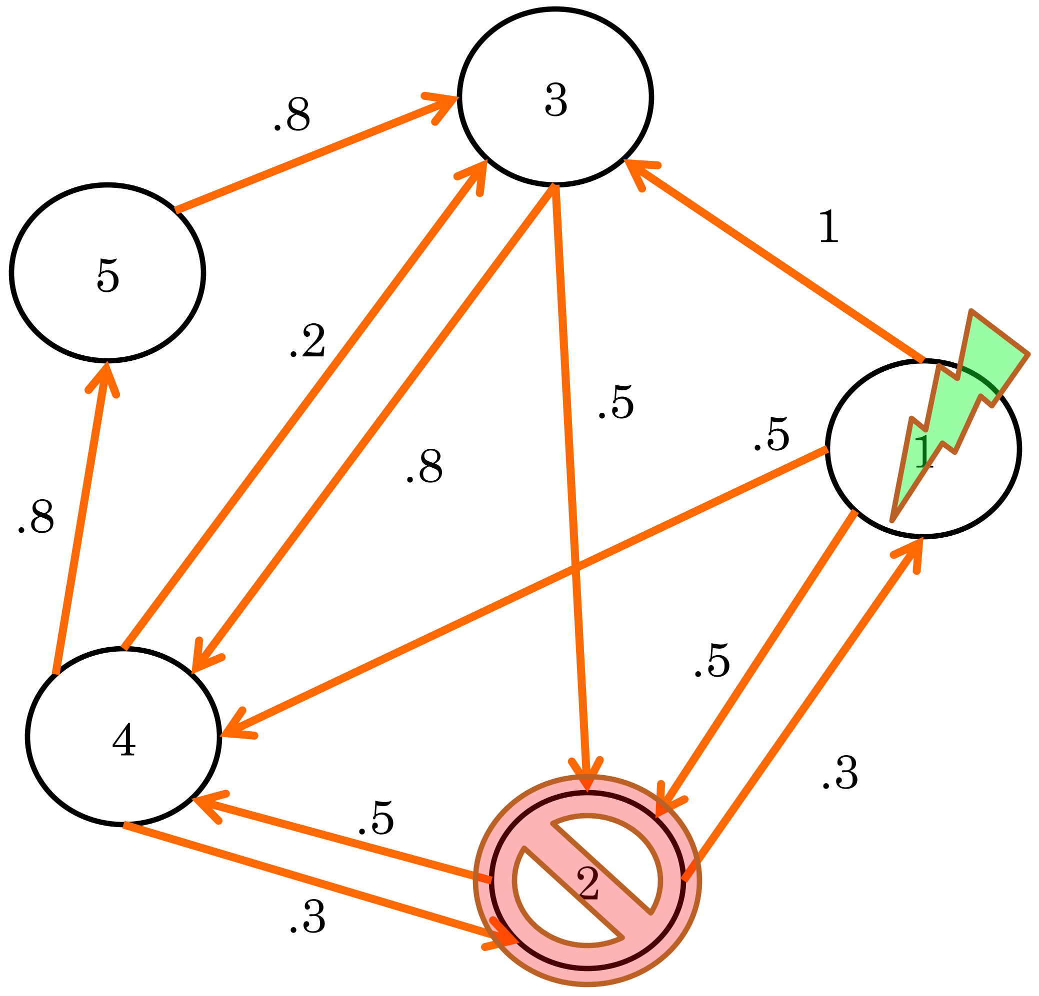
What the Learner Sees



Activations and Suppressions



ACTIVATIONS AND SUPPRESSIONS



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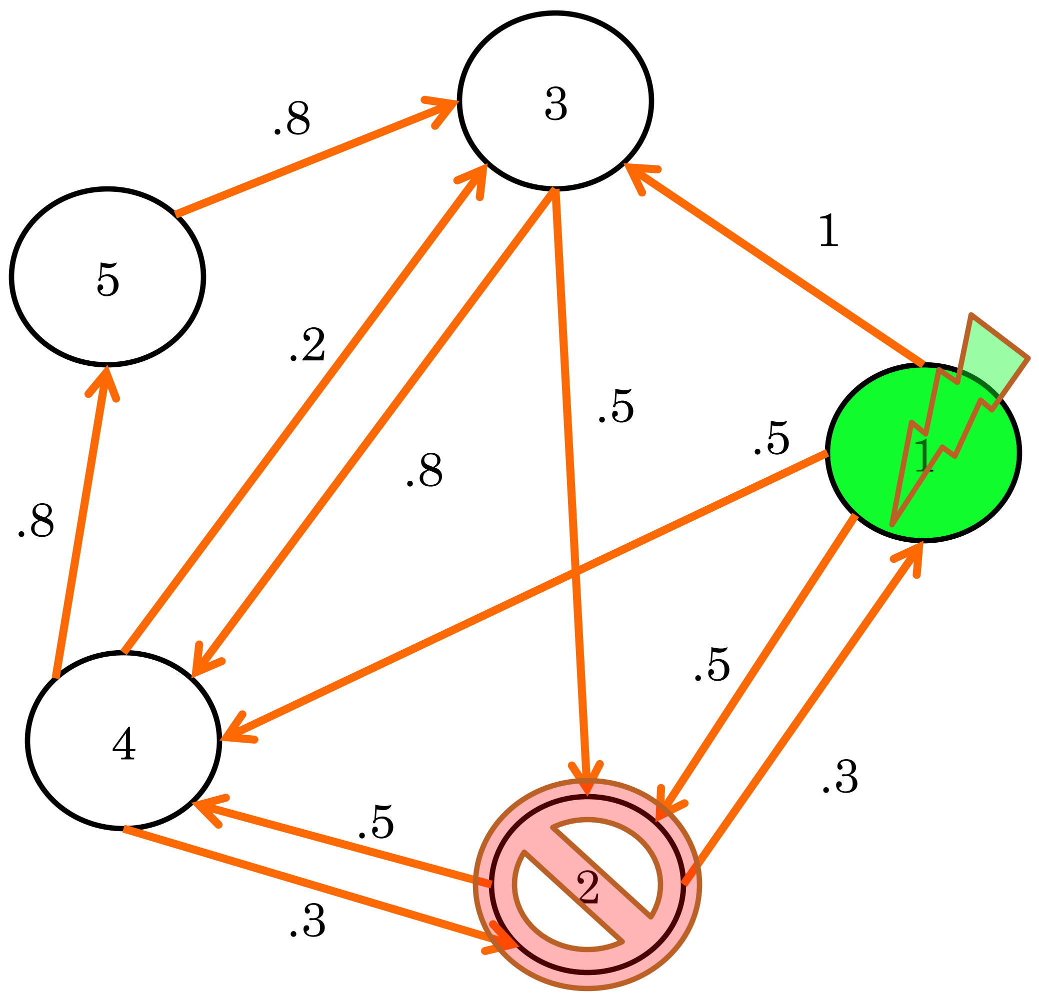

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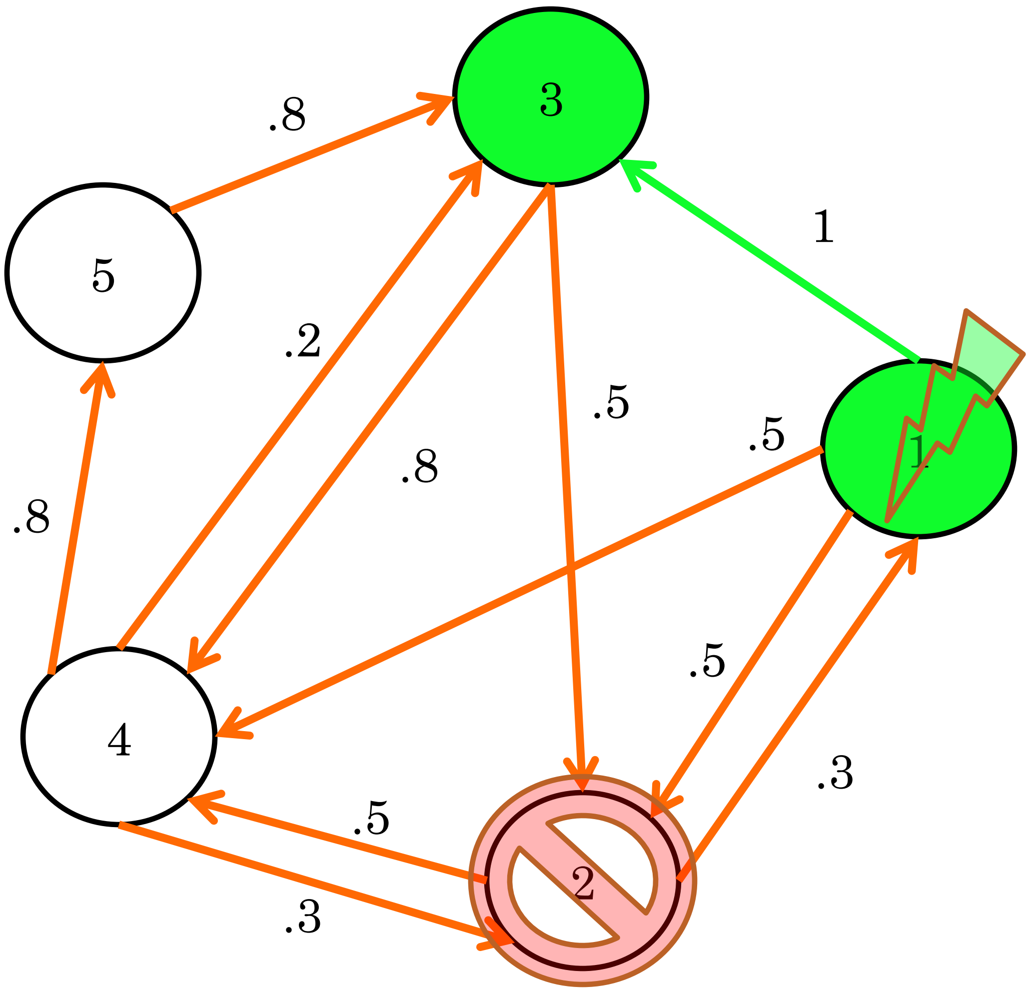

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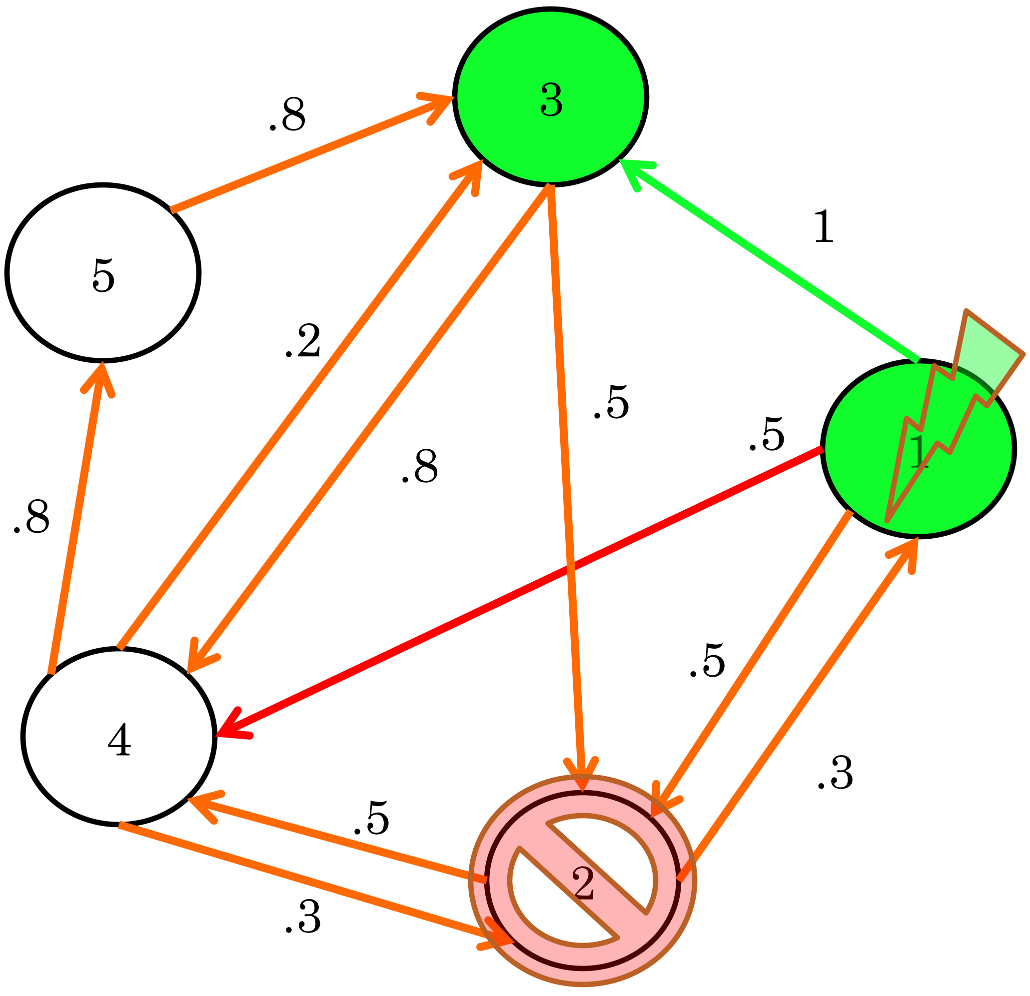

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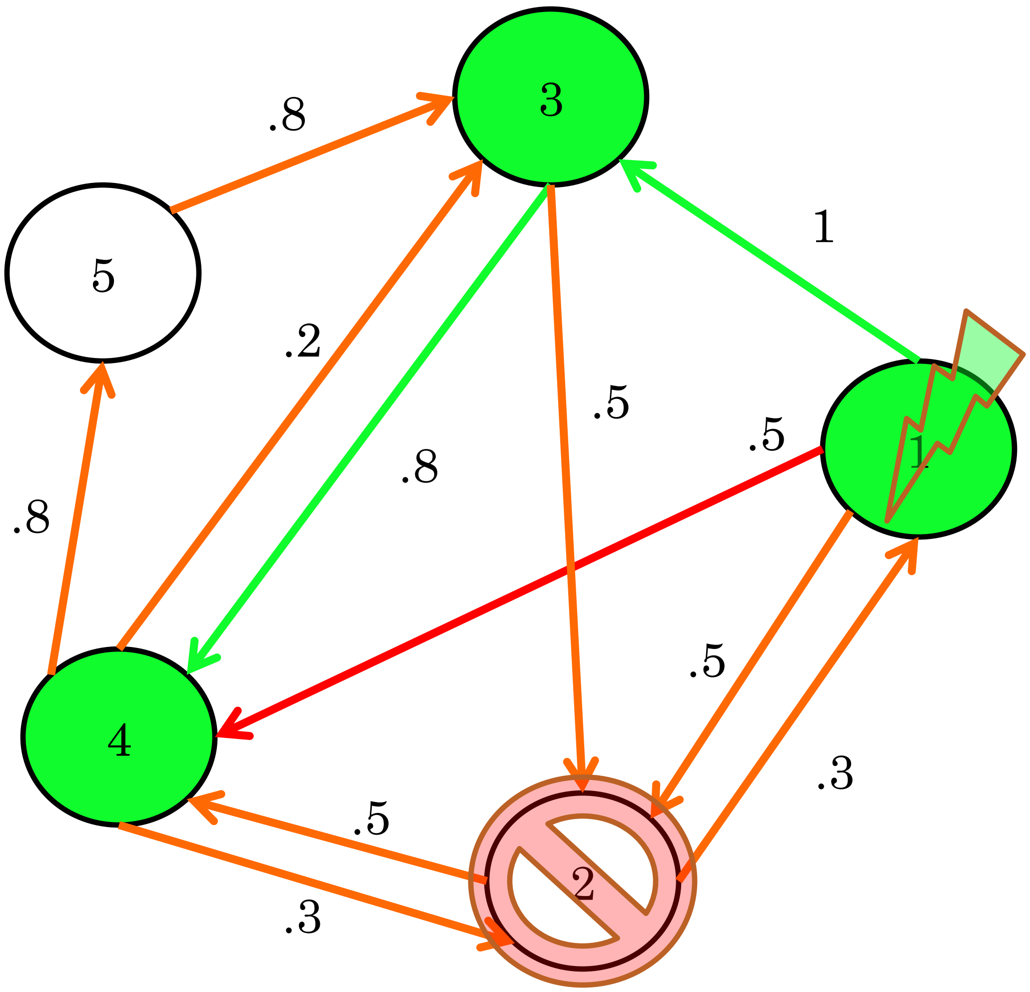

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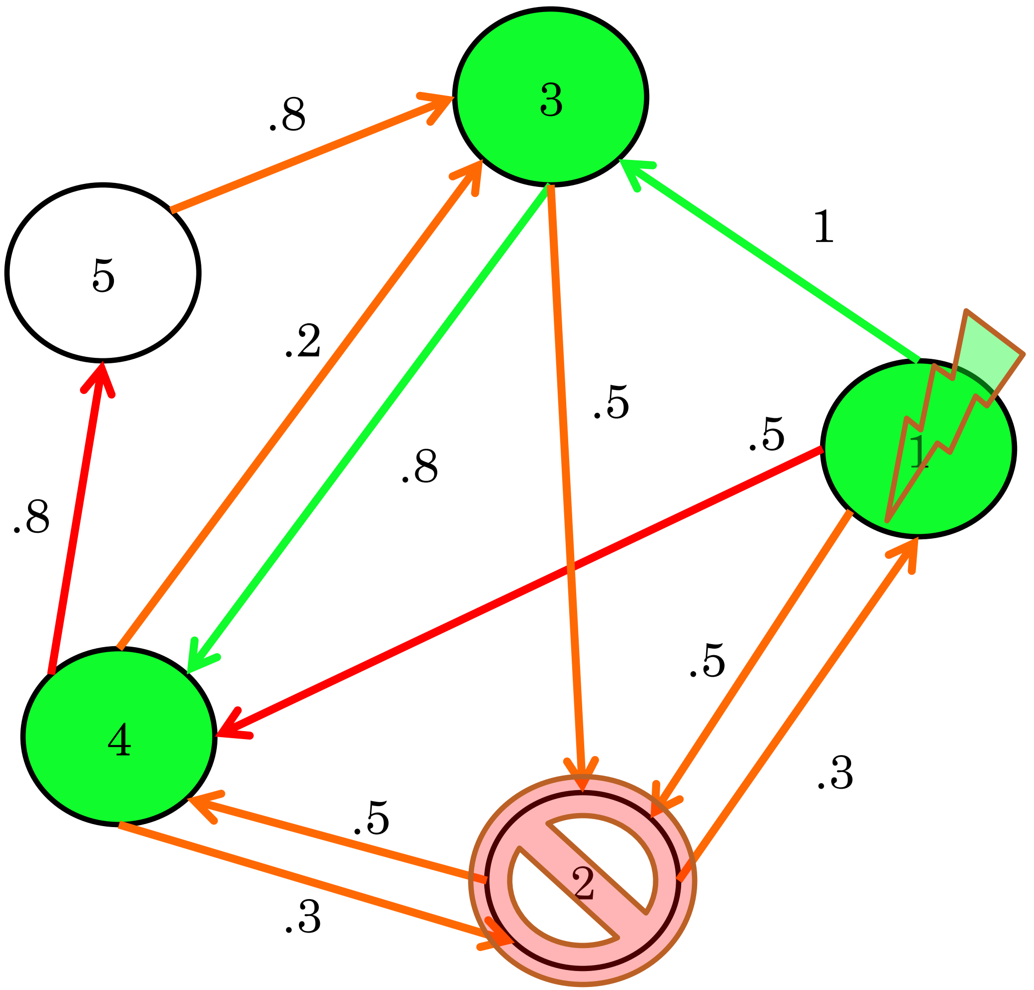

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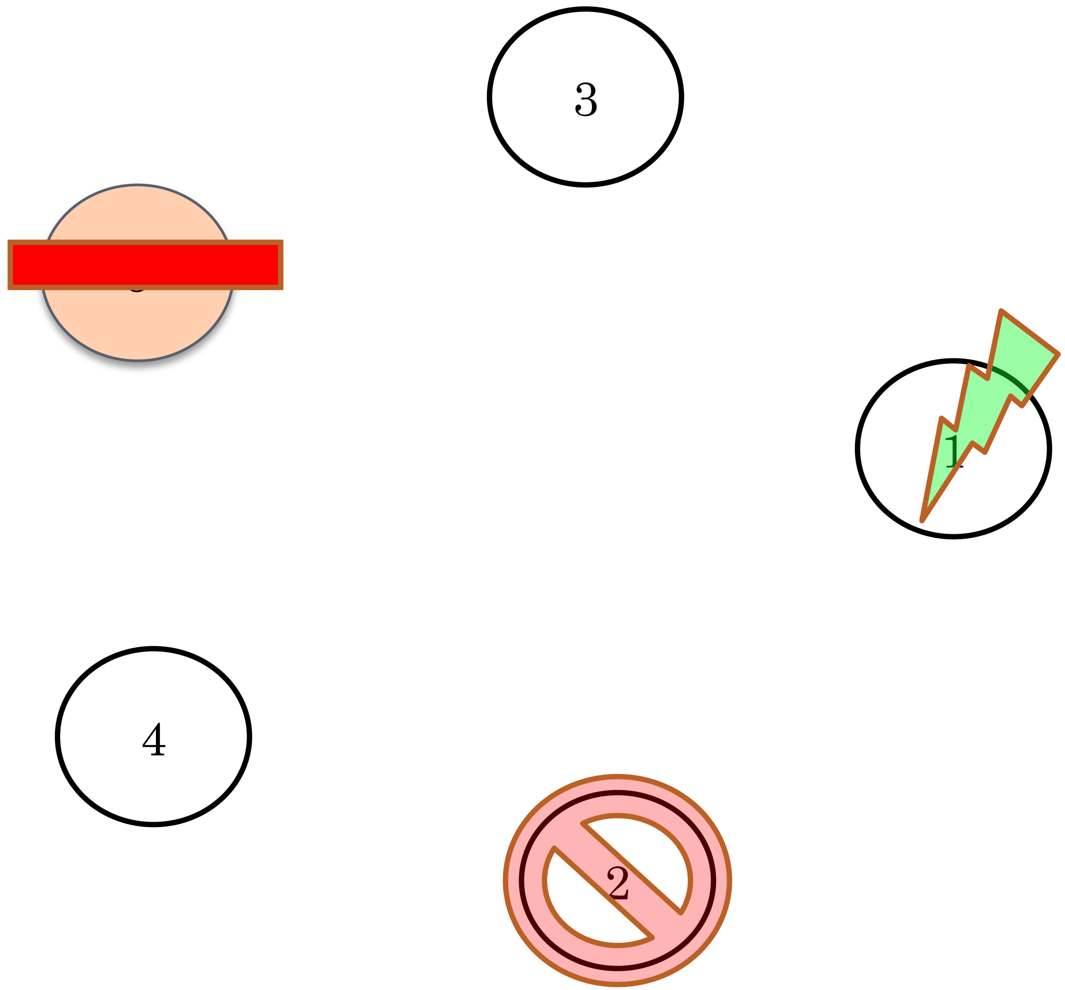

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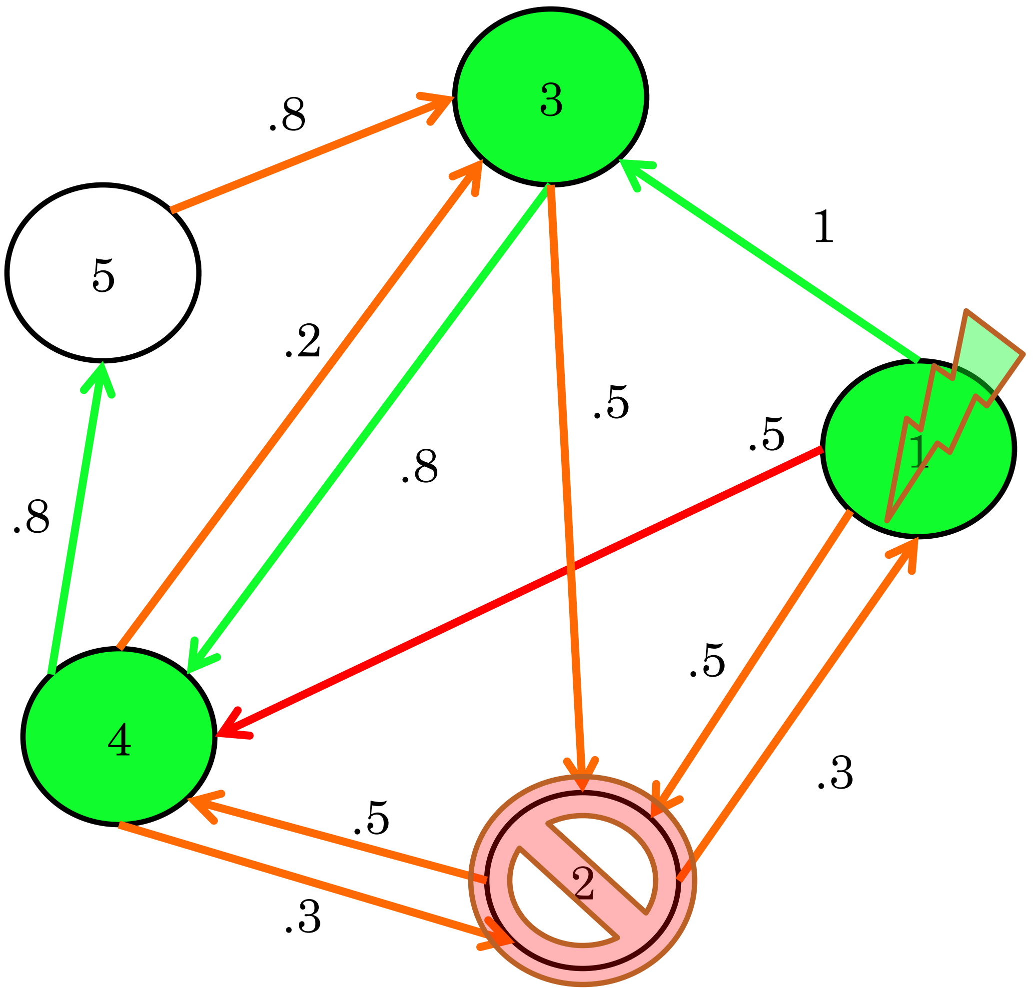

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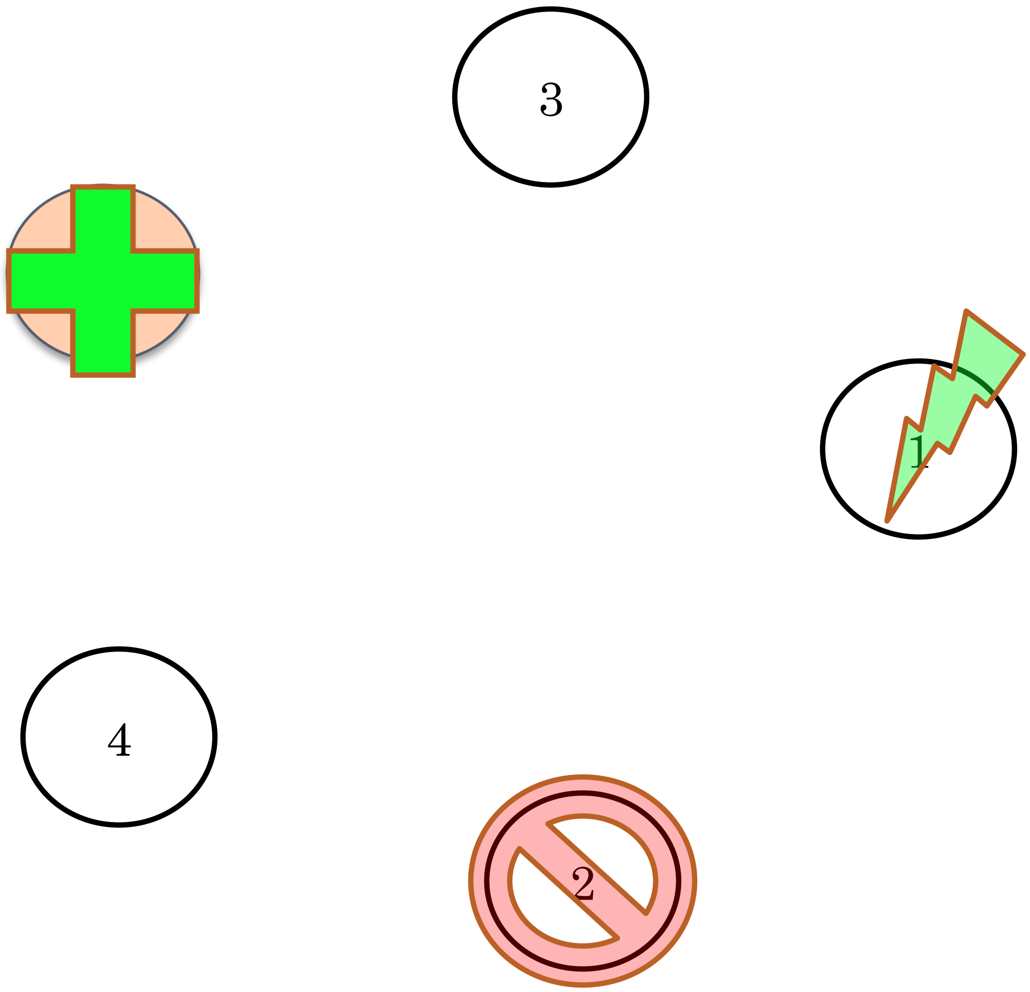
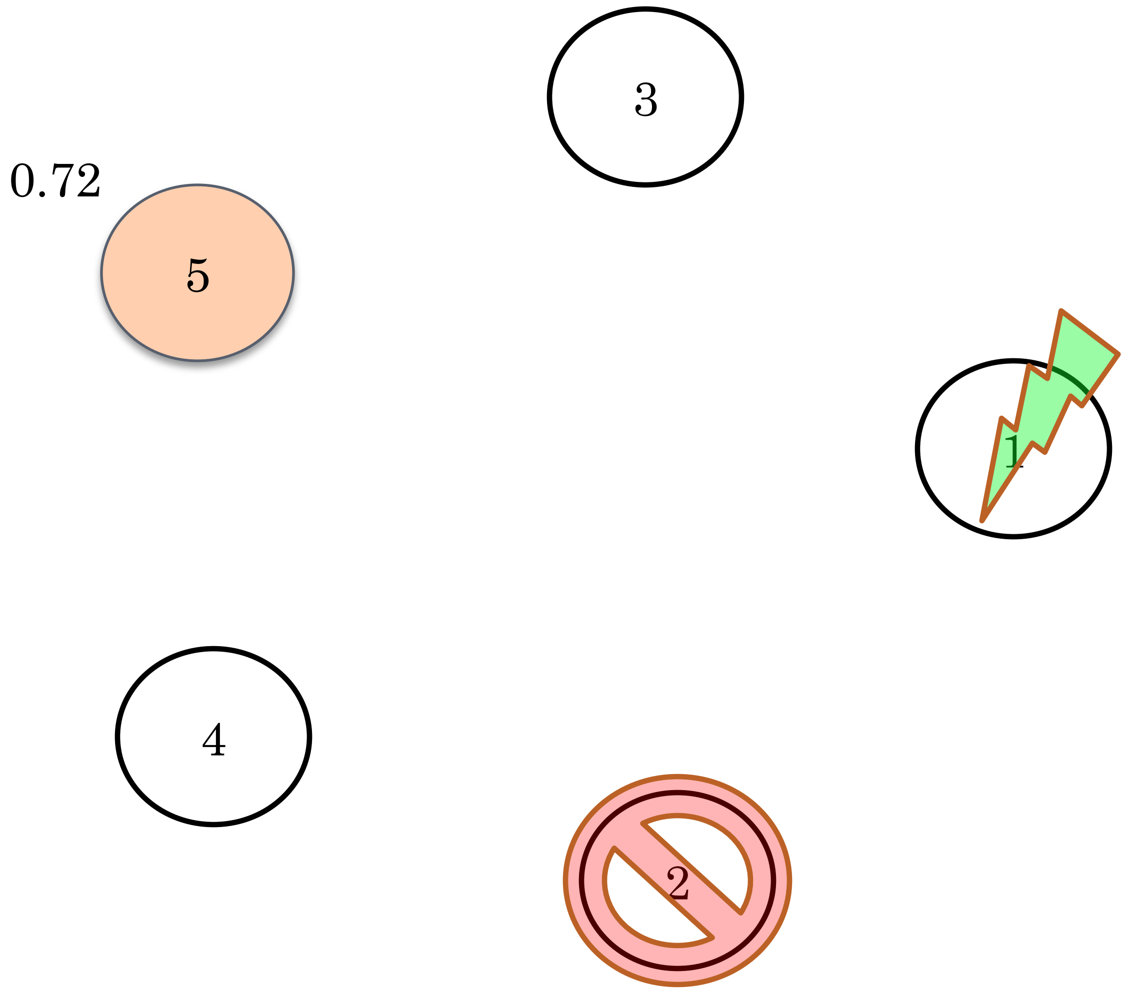


Exact Value Injection Queries



The Learning Task (a reminder)

- Two social networks S and S' are behaviorally equivalent if for any experiment e, S(e) = S'(e)
- Give access to a hidden social network S*, the learning problem is to find a social network S behaviorally equivalent to S* using value injection queries.

The Percolation Model

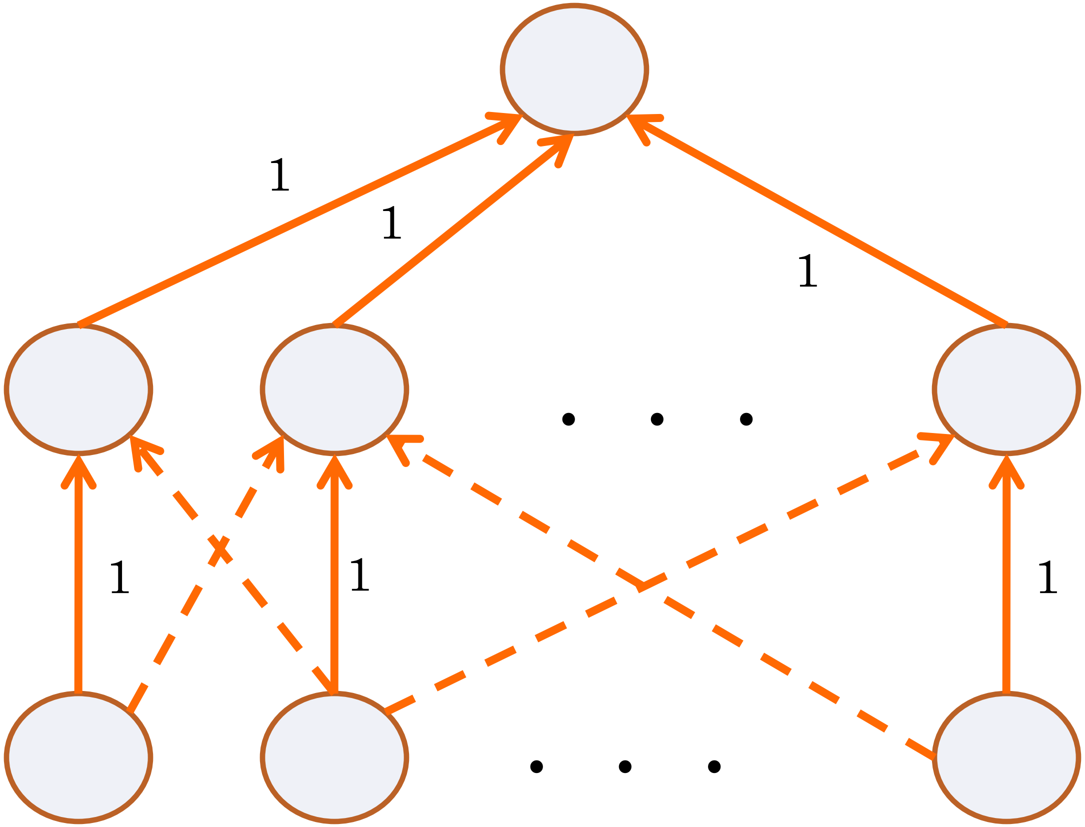
Given a network S and a VIQ

- All edges entering or leaving a suppressed node are automatically "closed."
- Each remaining edge (u,v) is "open" with probability $p_{(u,v)}$ and "closed" with probability (1- $p_{(u,v)}$)
- The result of a VIQ is the probability there is a path from a fired node to the output via open edges in S

Discoverable Edges

- Let S be a social network and S' be another social network that differs from S only in edge (u,v).
- We say edge (u,v) is discoverable if there is an experiment e such that $S(e) \neq S'(e)$.
- We can view the learning problem as having to find all discoverable edges.

A Lower Bound



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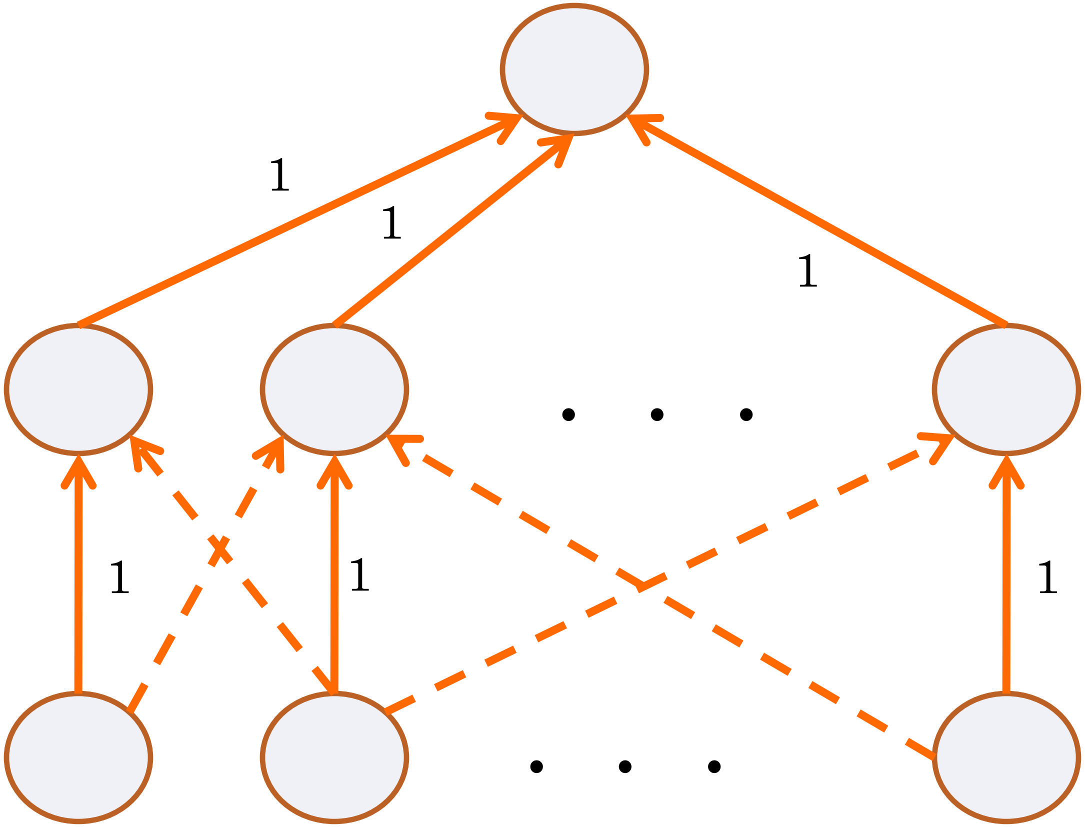
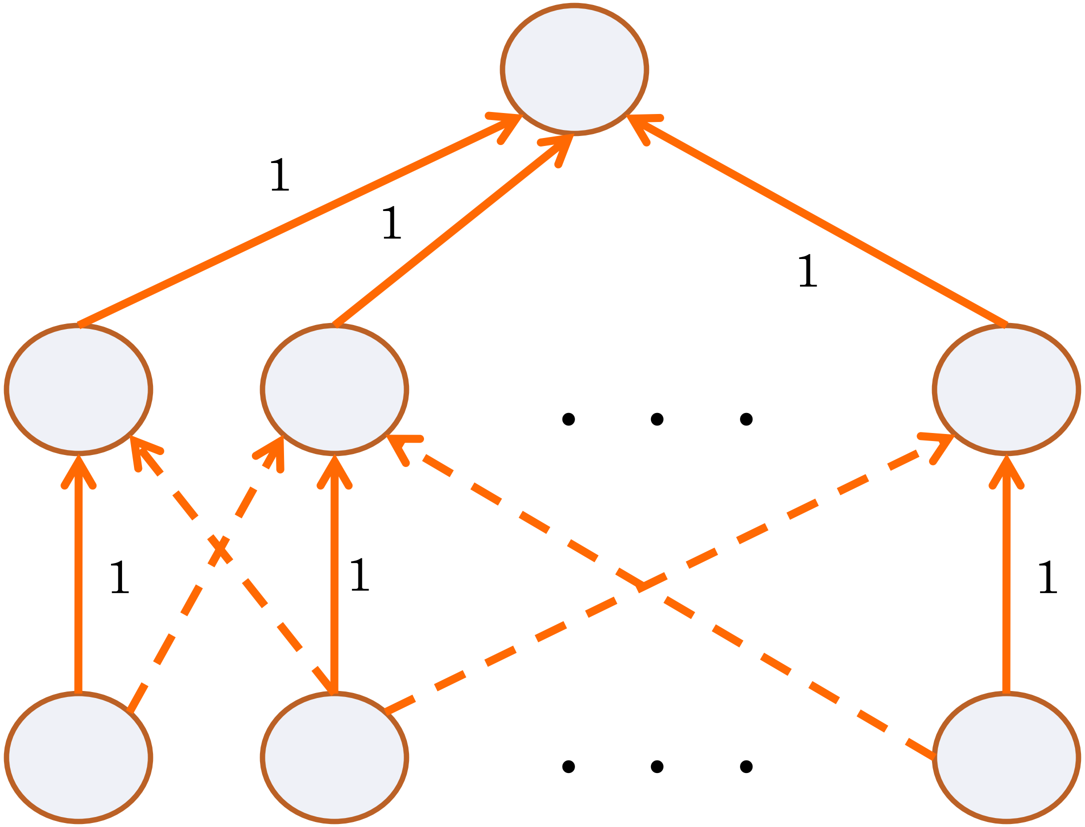


All queries give 1-bit answers

A Lower Bound



$2^{\Omega(n^2)}$ such graphs, $\Omega(n^2)$ l.b.

First Some Definitions

- The depth of a node is its distance to the root
- An Up edge is an edge from a node of larger depth to a node of smaller depth
- A Level edge is an edge between two nodes of same depth
- A Down edge is an edge from a node at smaller depth to a node at higher depth
- A leveled graph of a social network is the graph of Up edges

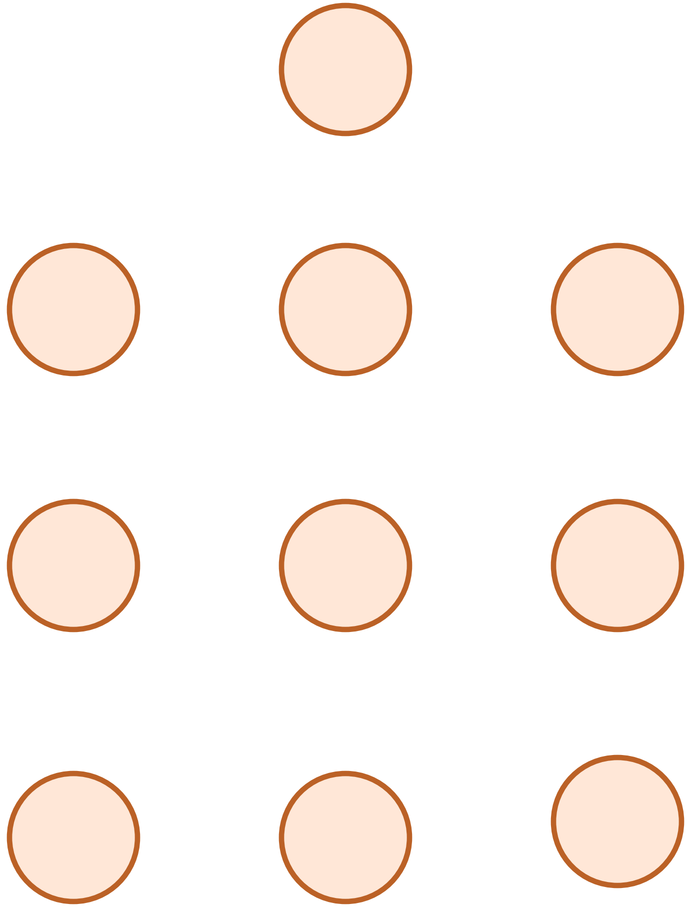
Excitation Paths

- An excitation path for a node n is a VIQ in which a subset of the free agents form a simple directed path from n to the output. All agents not on the path with inputs into the path are suppressed.
- We also have a shortest excitation path

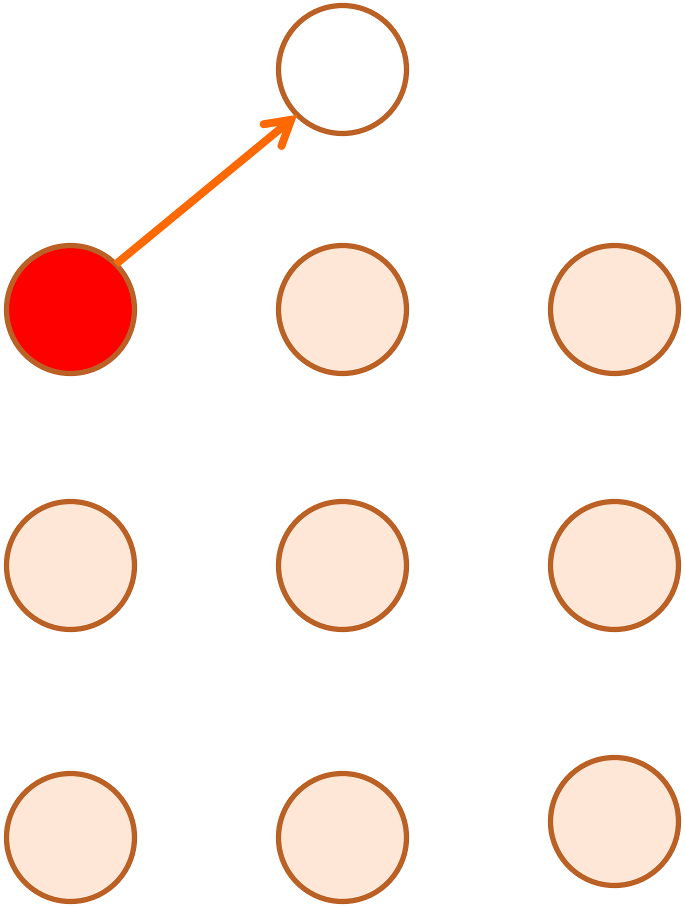
The Learning Algorithm For Networks Without 1 Edges

- First Find-Up-Edges to learn the leveled graph of S
- For each level, Find-Level-Edges
- For each level, bottom-down, Find-Down-Edges

Find-Up-Edges



Find-Up-Edges



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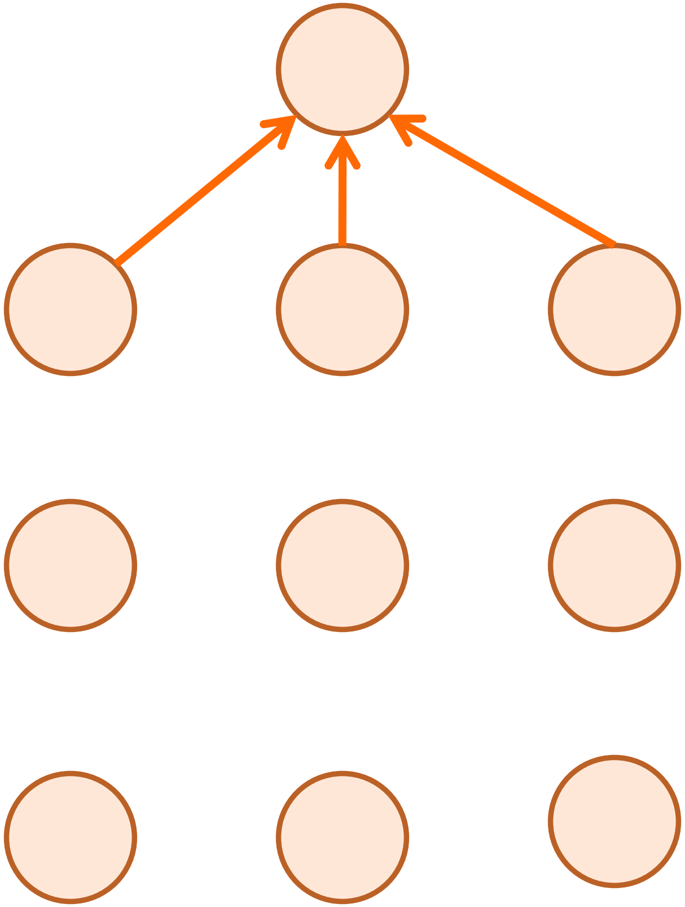

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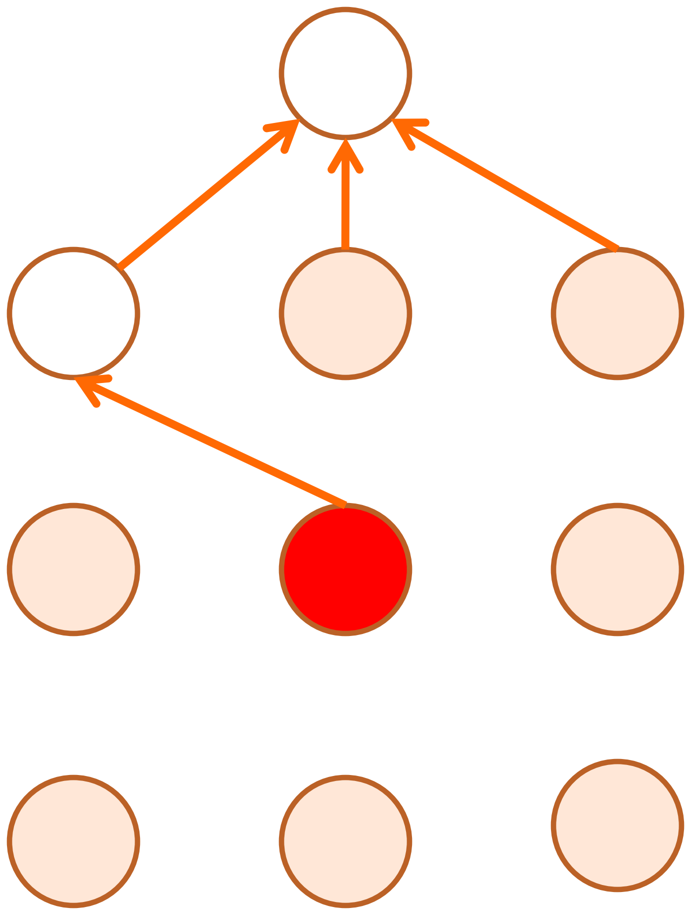
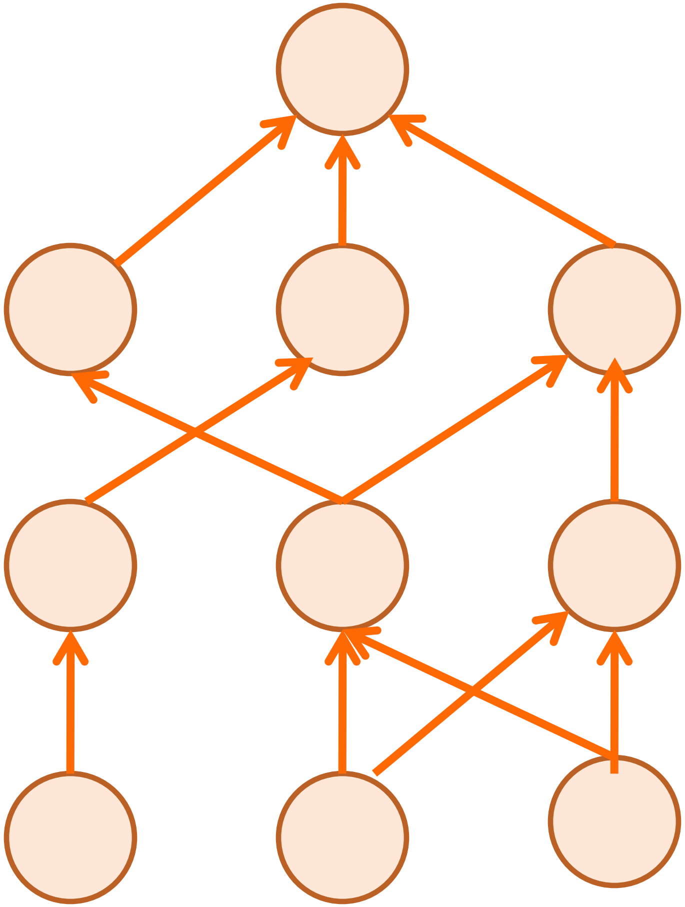
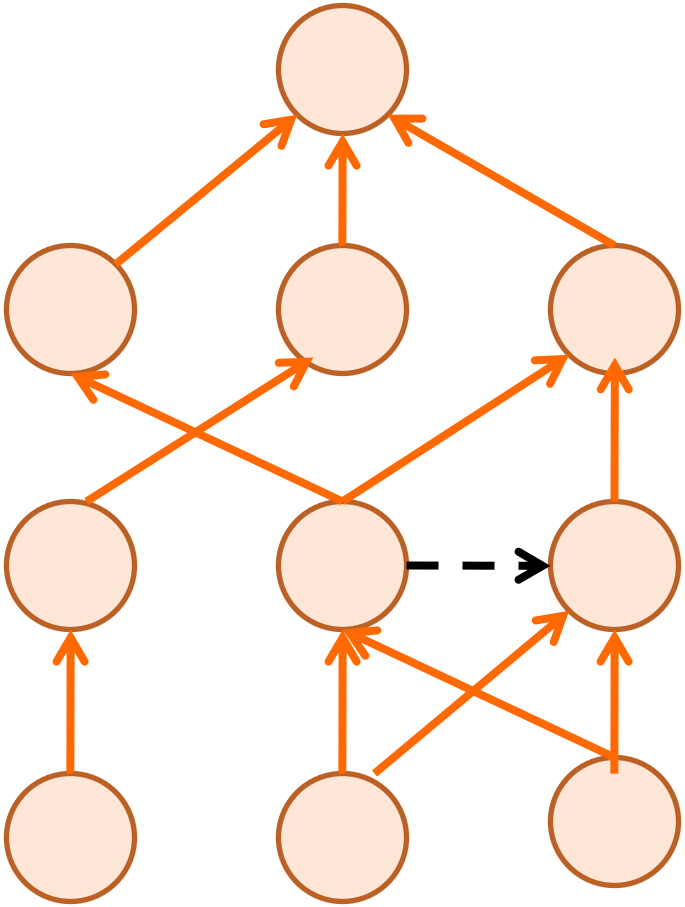


FIND-UP-EDGES



Find-Level-Edges



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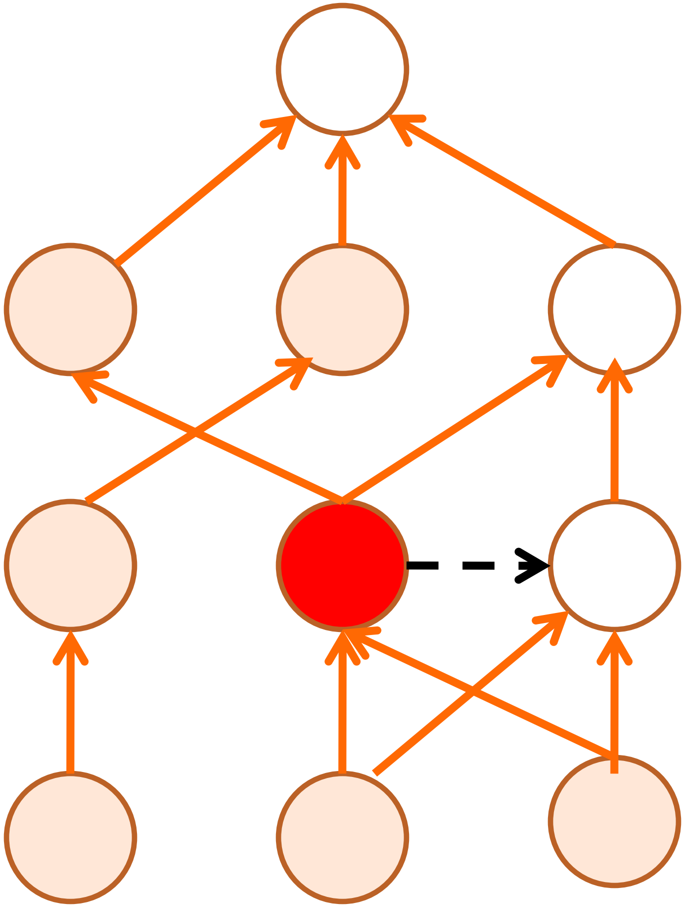

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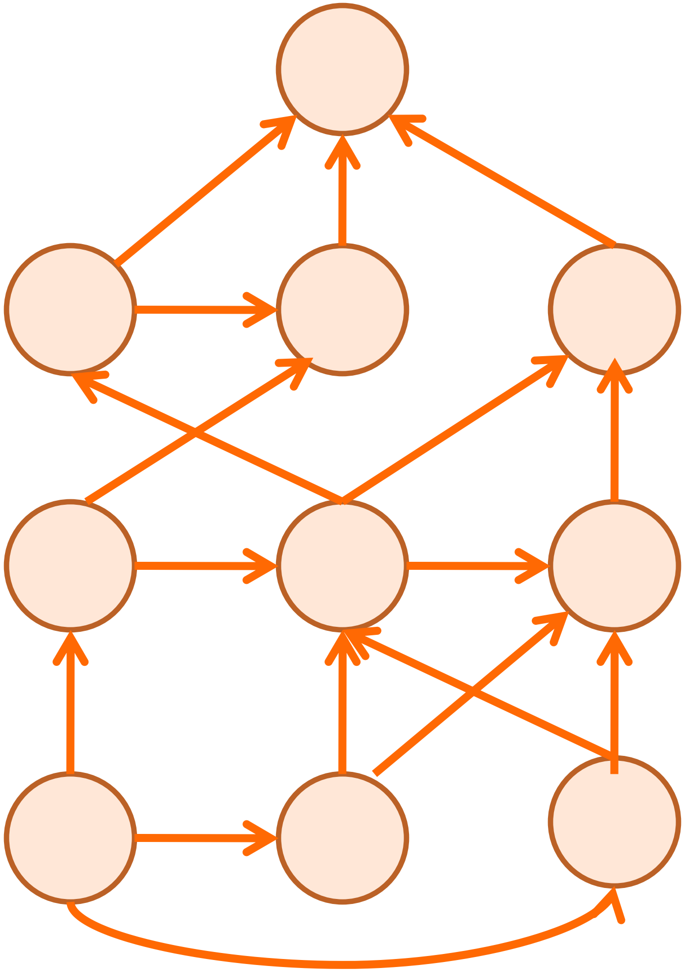
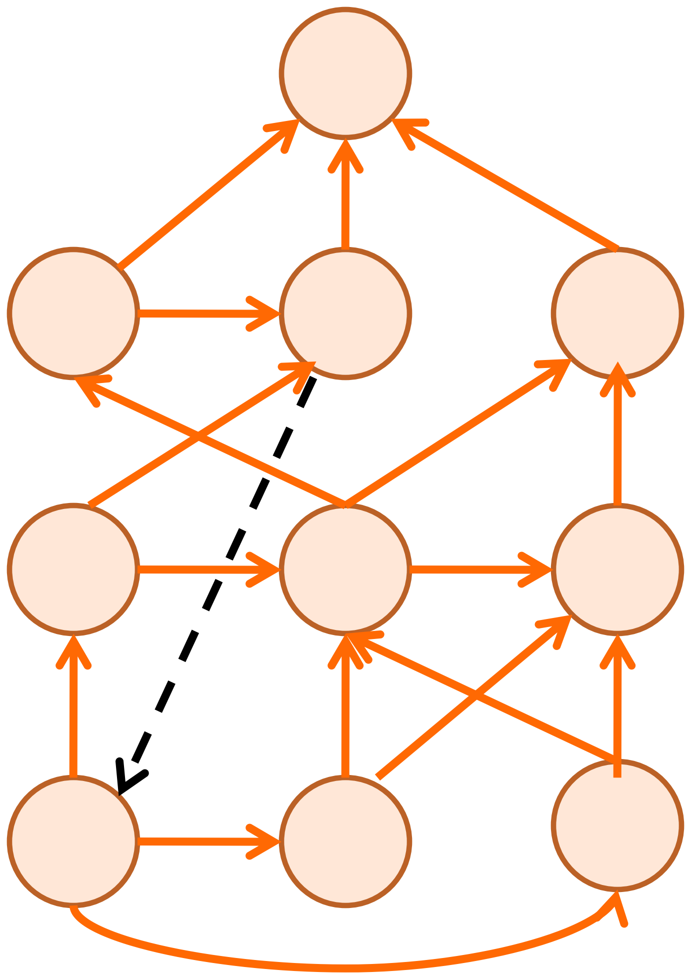


FIND-DOWN-EDGES



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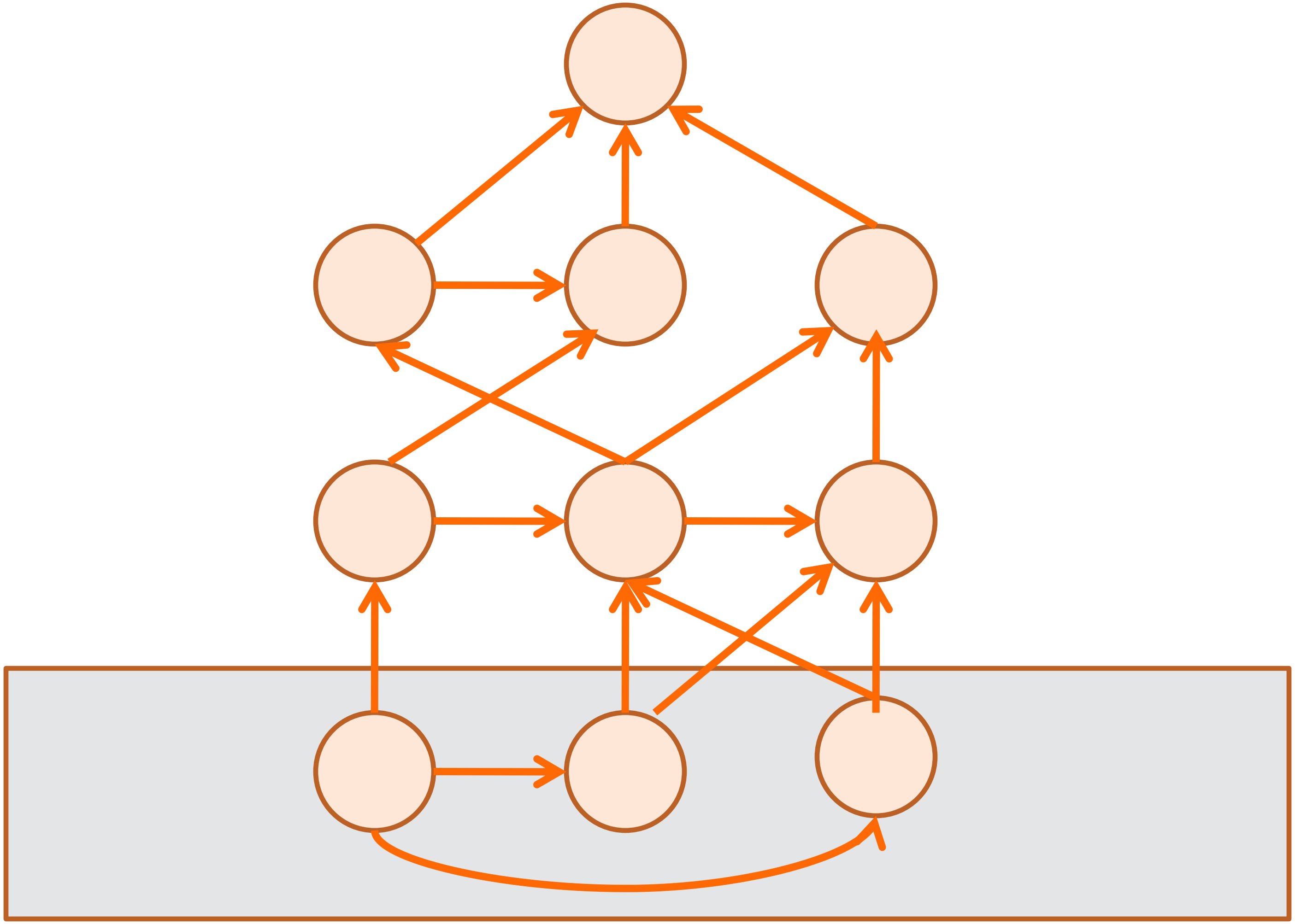

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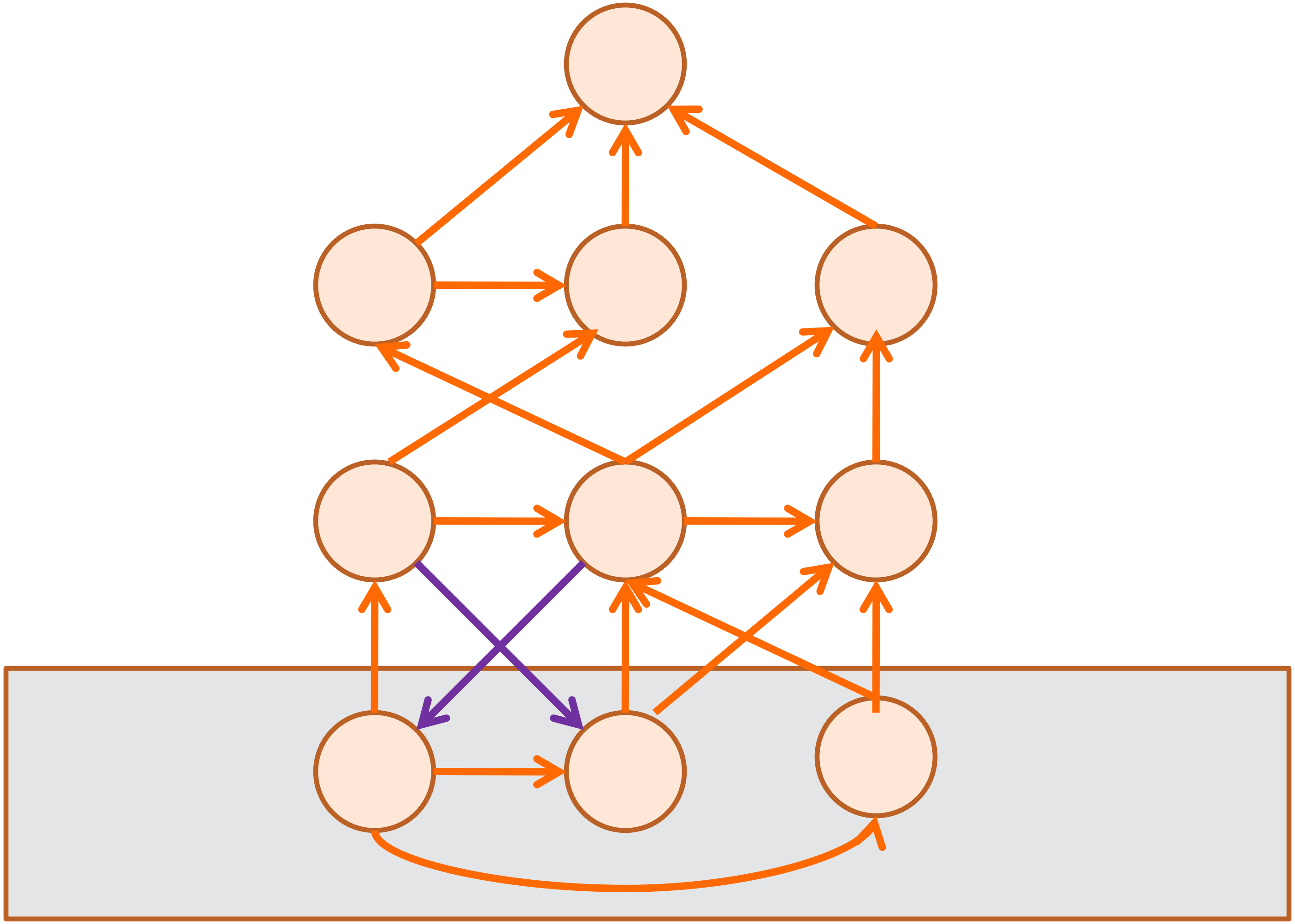

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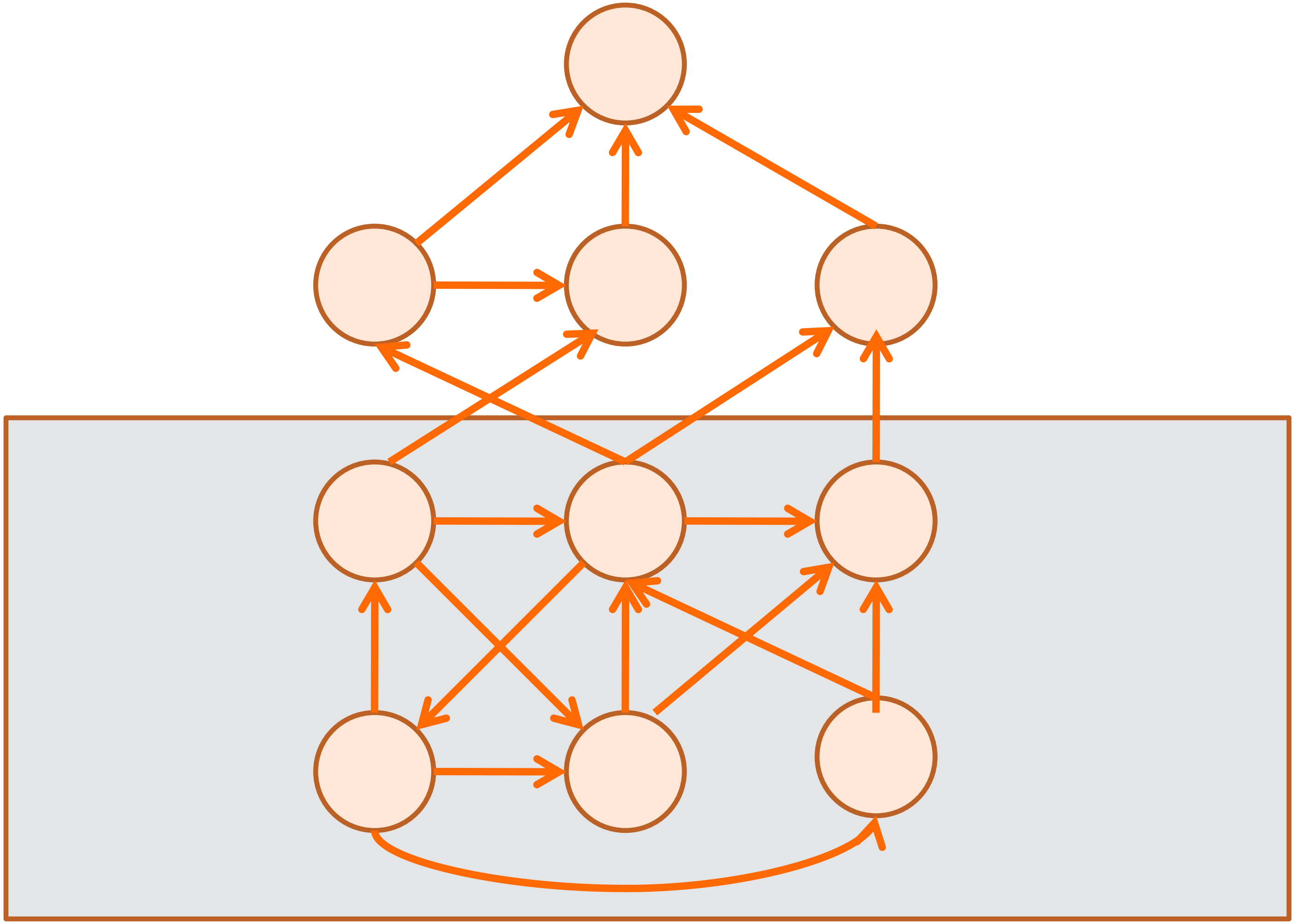

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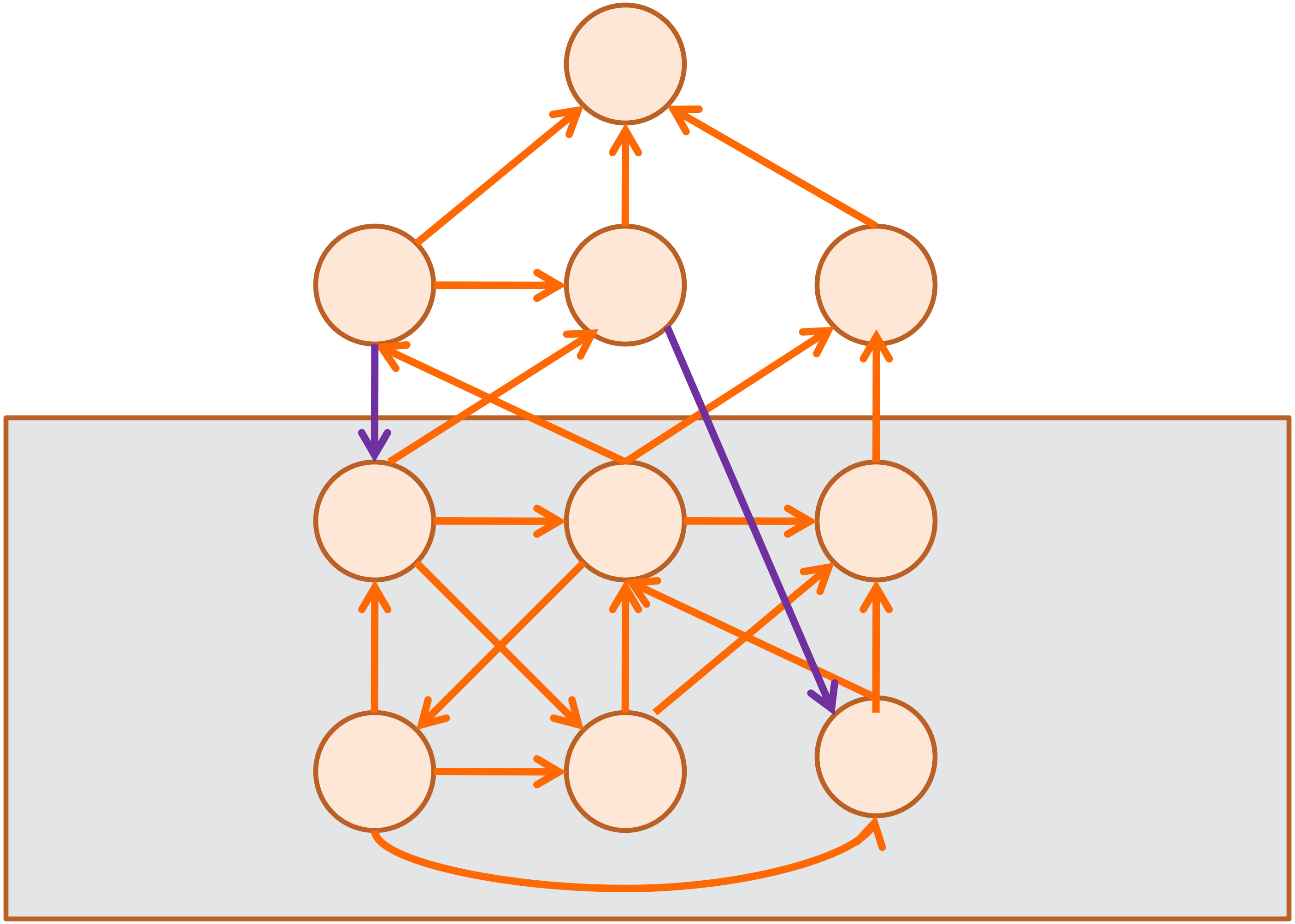

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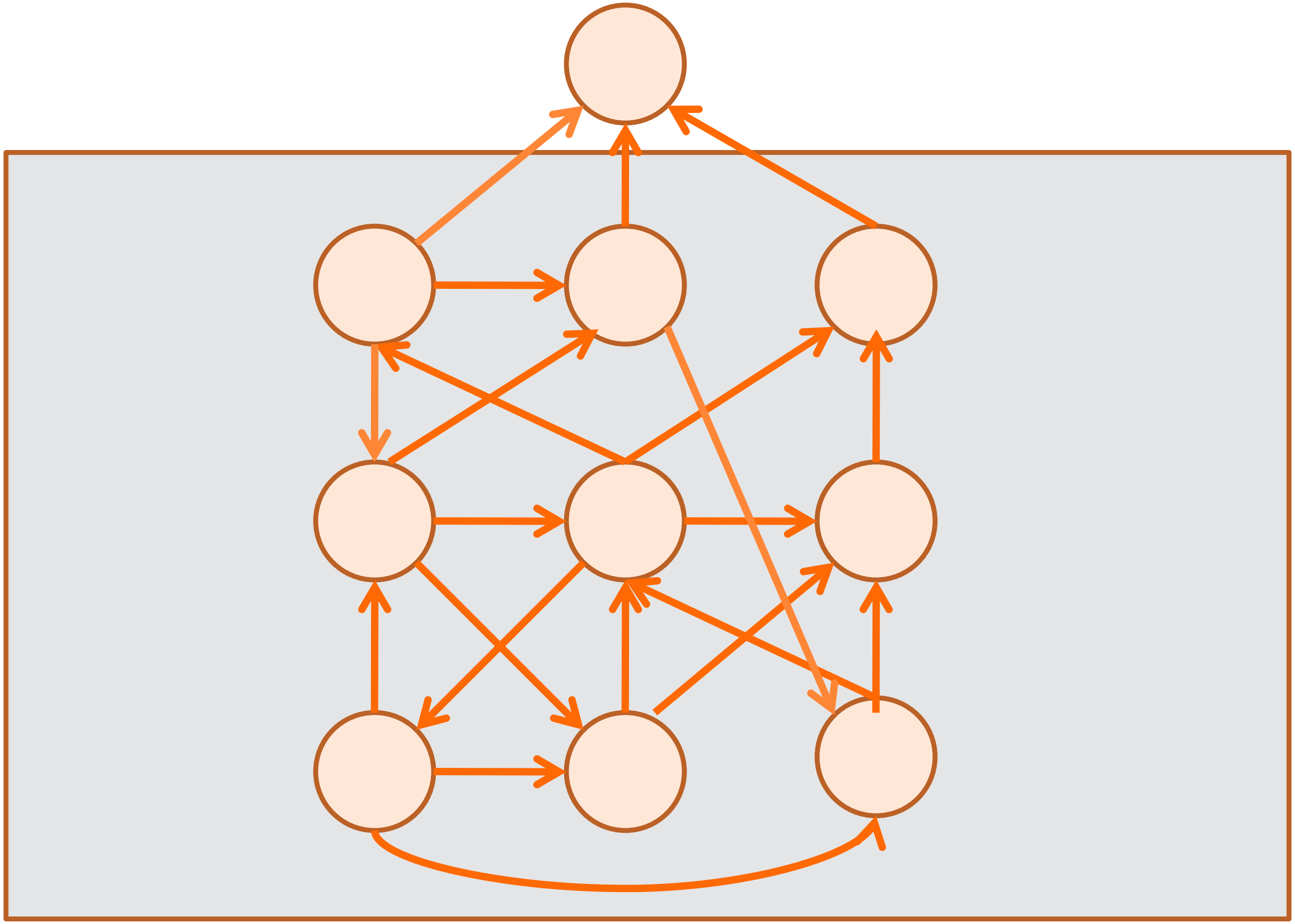

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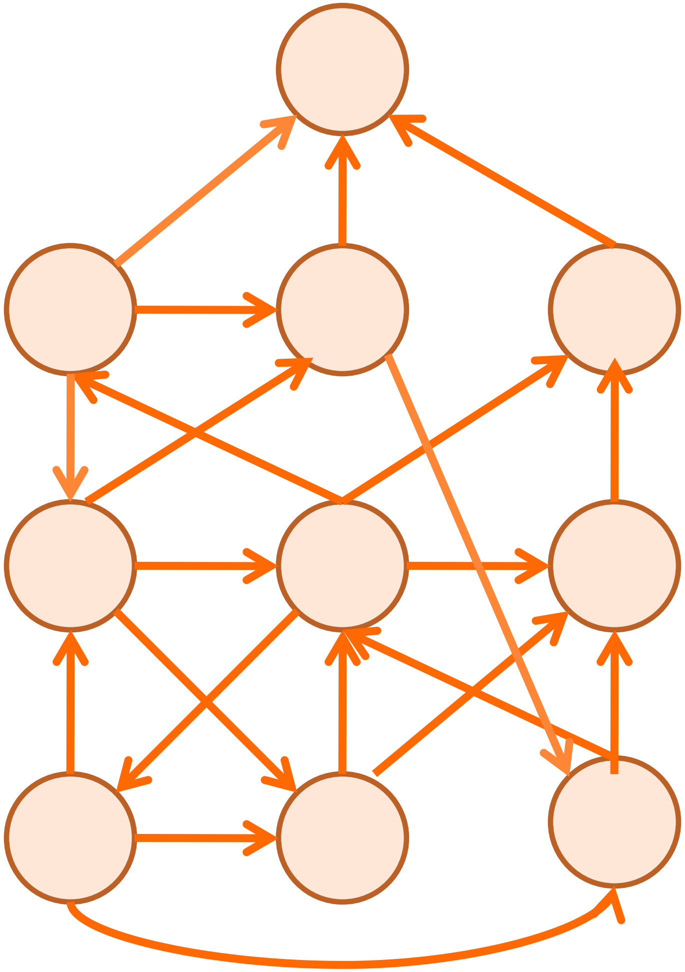
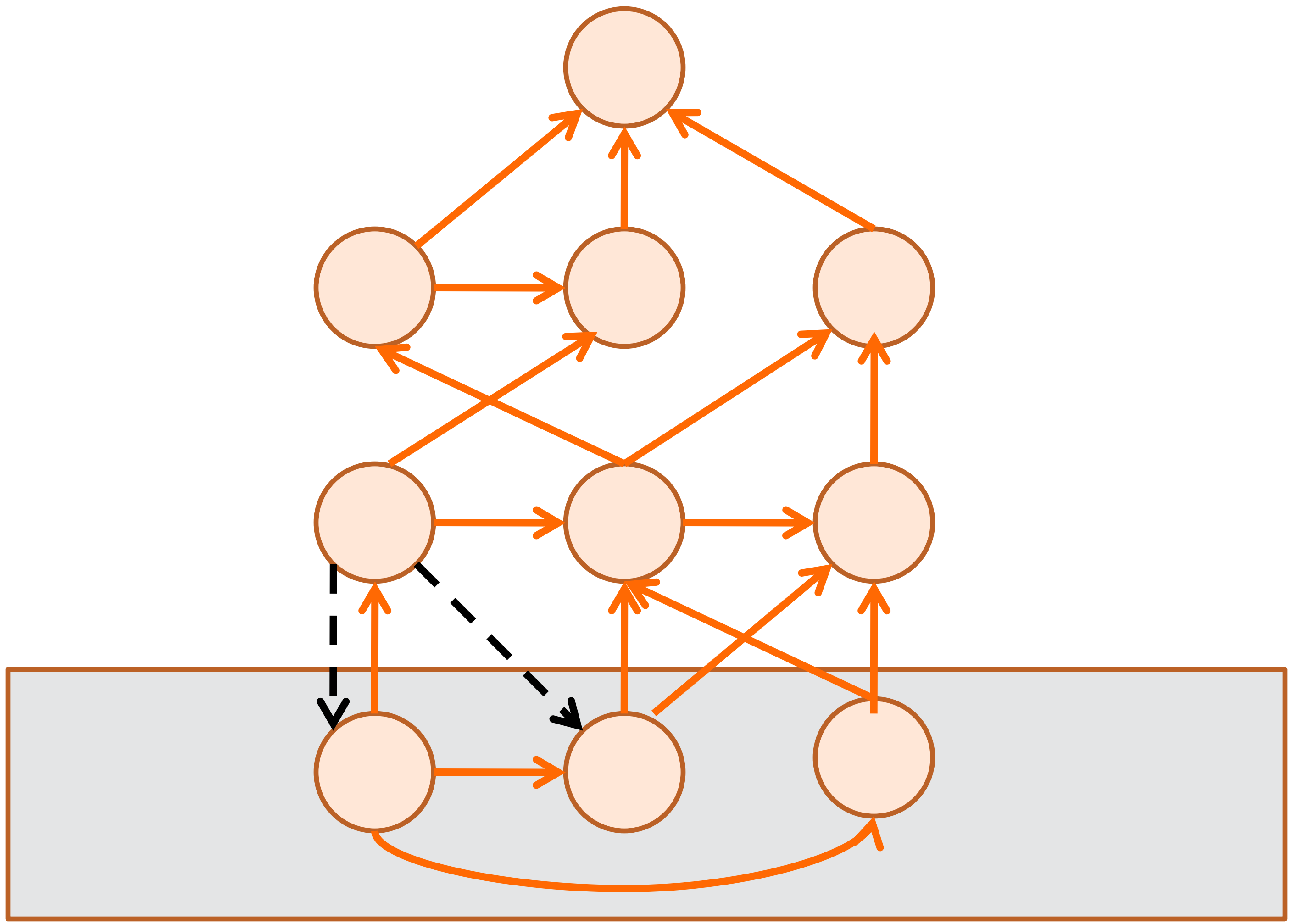


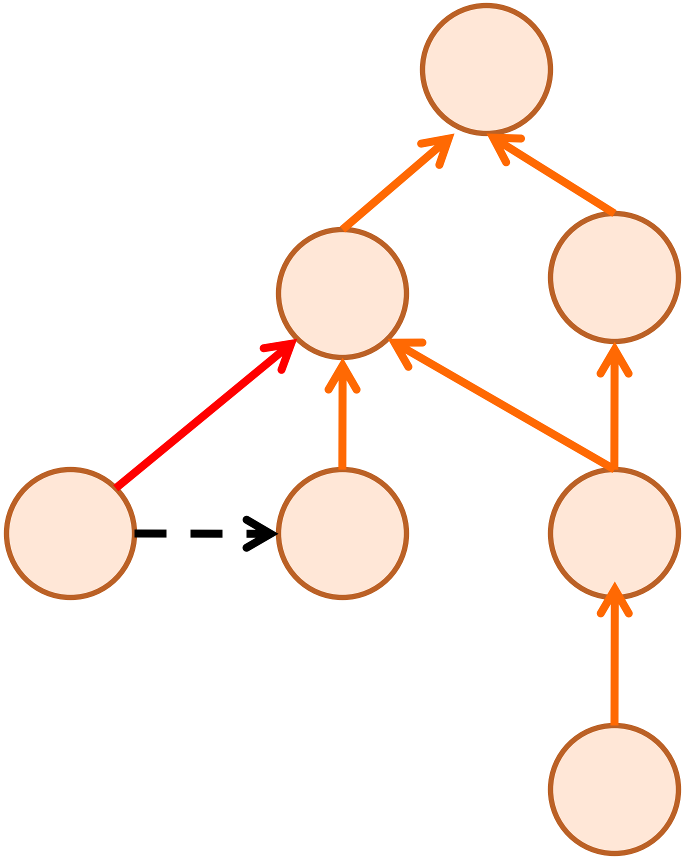
Find-Down-Edges

- For each node u at current level
  - Sort each node v_i in C (complete set) by distance to the root in $G - \{u\}$
  - Let $v_1 \ldots v_k$ be the sorted v_is
  - Let $pi_1 \ldots pi_k$ be their corresponding shortest paths to the root in $G - \{u\}$
  - For i from 1 to k
    - Do experiment of firing u, leaving pi_i free, and suppressing the rest of the nodes.

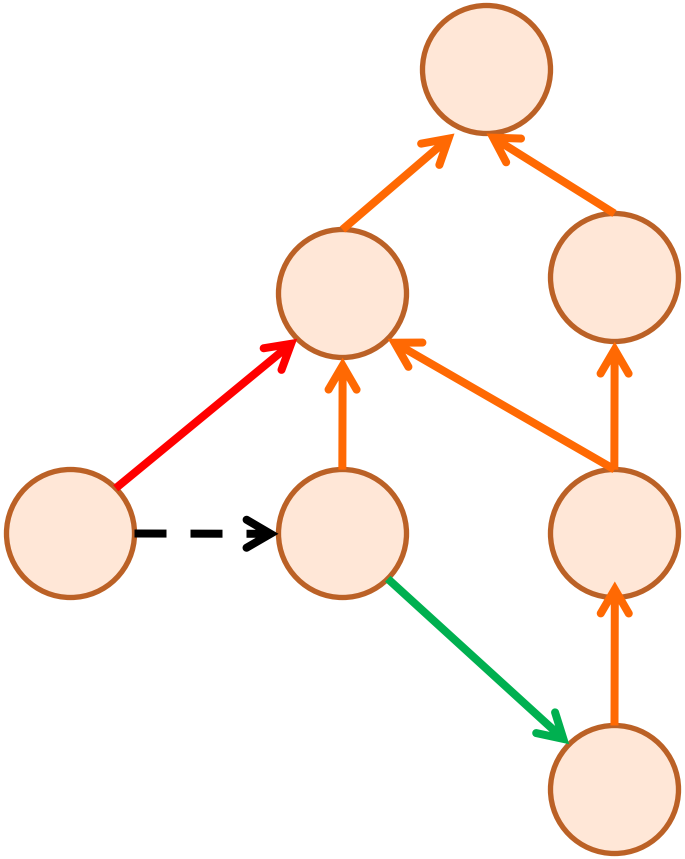
For Example



With Ones – a Problem



With Ones – A Problem



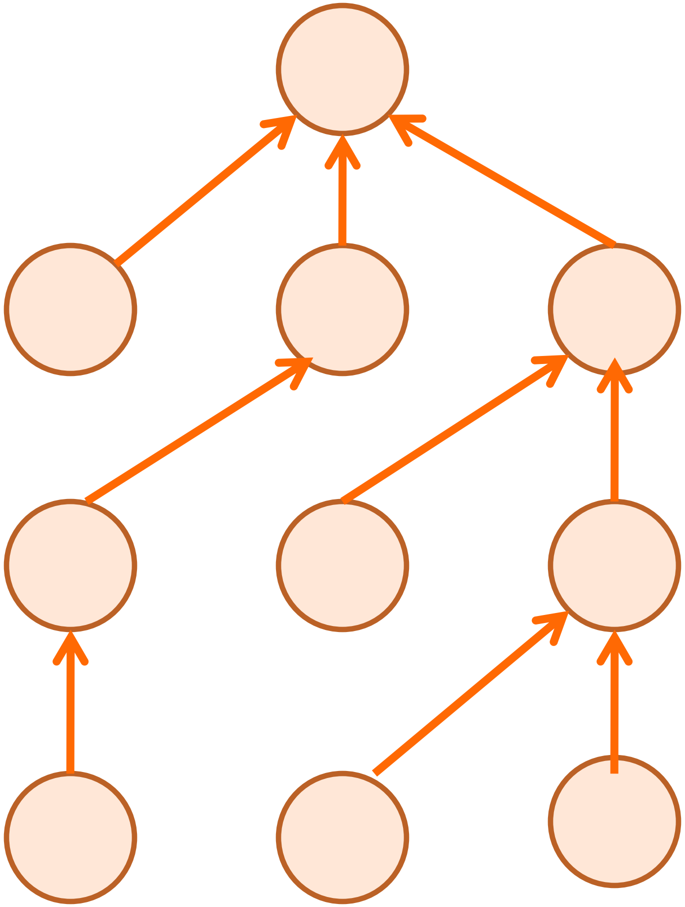
With Ones

- Algorithm gets more complicated
- Level edges and down edges are found in one subroutine
- In looking for down edges from u, need to avoid not just u, but also all nodes reachable from u by 1 edges
- There always exists some pair of nodes, with source in L (current level) and destination in C + L where you can look for an edge.

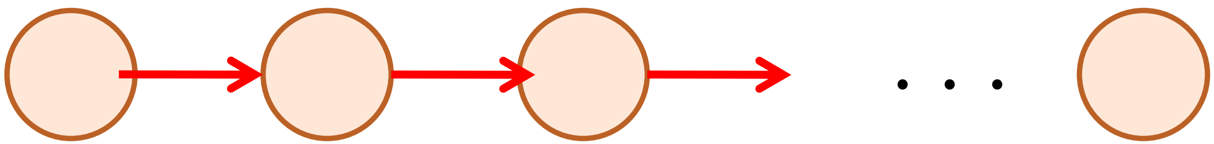
In the End

- We do 1 query per each possible edge, giving an $O(n^2)$ algorithm
- Matches the $\Omega(n^2)$ lower bound

Trees



Lower Bound



Like sorting with comparisons

Gives a $\Omega(n \log n)$ lower bound

Algorithm

- Ancestor Test. To test if u has an ancestor in set S, fire u and suppress all nodes in S.
- How to make a Parent Finder using log n Ancestor Tests and other queries? (assume no 1 edges)

Algorithm

- Ancestor Test. To test if u has an ancestor in set S, fire u and suppress all nodes in S.
- How to make a Parent Finder using log n Ancestor Tests and other queries? (assume no 1 edges)
- Remember, your parent is your deepest ancestor

Trees

- Using n queries find all distances to the root
- Using log n queries per node find all parents
- Then “sort out” all 1 edges
- Gives a O(n log n) algorithm that meets the lower bound

Finding Influential Nodes

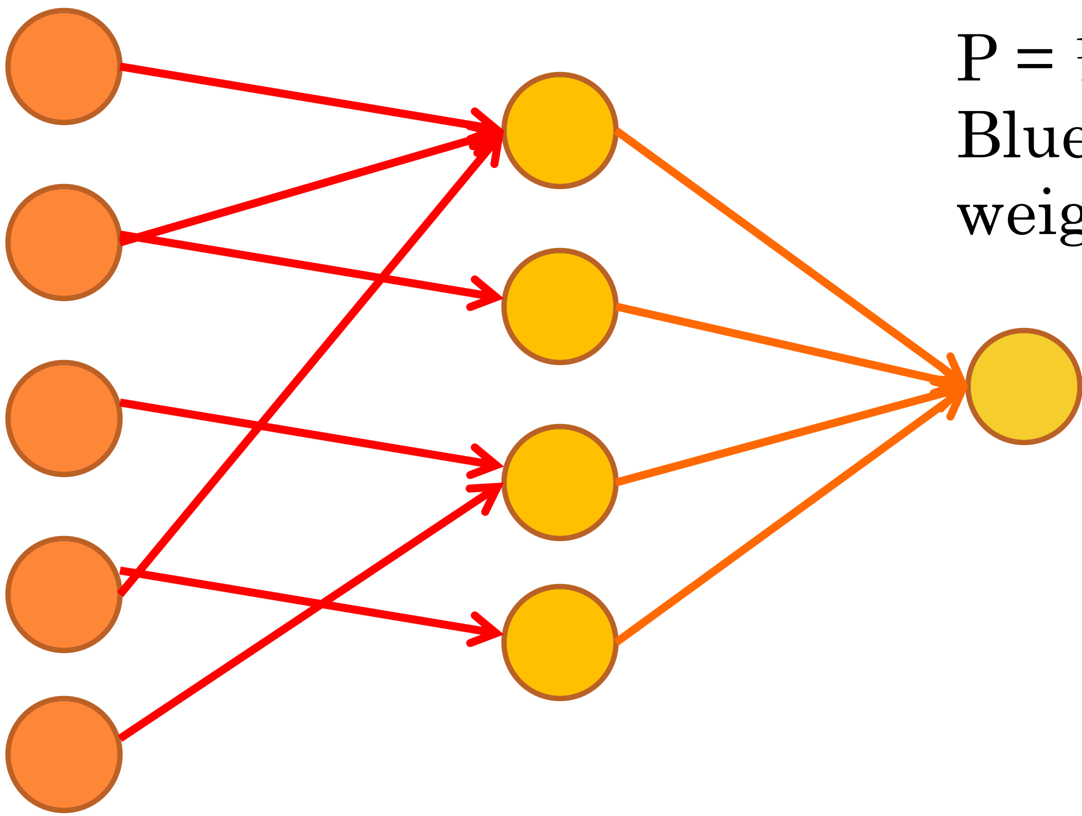
- Suppose instead of learning the social network, we wanted to find an influential set of nodes quickly.

- A set of nodes is influential if, when activated, activates the output with probability at least p

Finding Influential Nodes

- Suppose instead of learning the social network, we wanted to find an influential set of nodes quickly.
- A set of nodes is influential if, when activated, activates the output with probability at least p
- NP Hard to Approximate to log n, even if we know the structure of the network

Reduction from Set Cover



P = ½
Blue edges have weight (1- ½ ^(1/k))

sets k elements

An Approximation Algorithm

- Say the optimal solution has m nodes
- Suppose we wanted to fire the output with probability (p – ε)
- Let I be the set of chosen influential nodes.
- Observation: at any point in the algorithm, greedily adding one more node w to I makes

$$S(e_{I\cup\{w\}}) \geq S(e_I) + \frac{p - S(e_I)}{m}$$

ANALYZING GREEDY

- Using a greedy algorithm, we let k be the number of rounds the algorithm is run

For

$$p\left(1-\frac{1}{m}\right)^k < \epsilon$$

it suffices that

$$e^{-\frac{k}{m}} < \frac{\epsilon}{p}$$

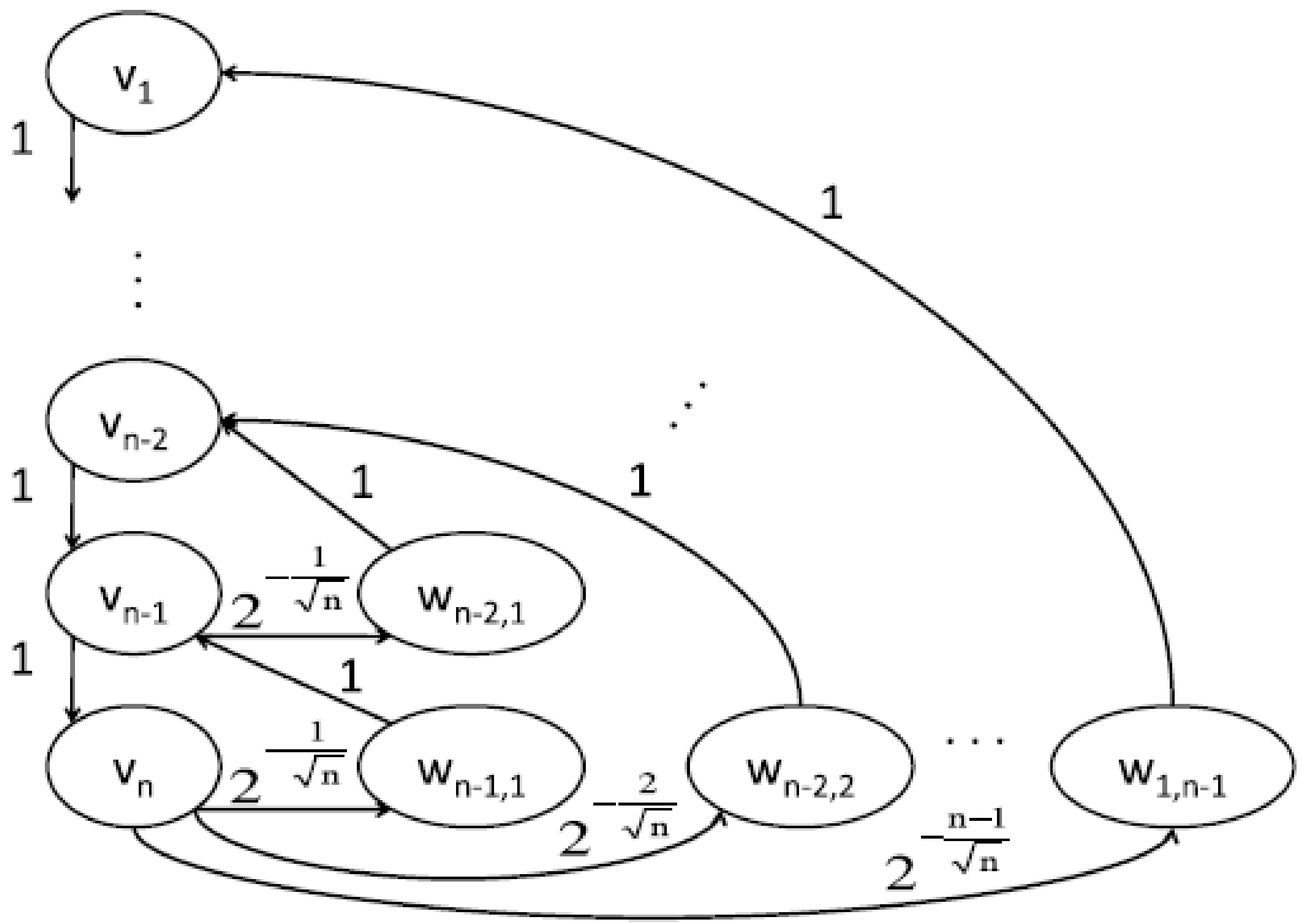
or

$$k > m\log\left(\frac{p}{\epsilon}\right).$$

Reconciling Greedy

- Therefore after $m \log(p/\varepsilon)$ rounds, we get to within ε of p.
- To reconcile with the set cover reduction, if we set $\varepsilon = ½^\wedge(1/n) - ½^\wedge(1/(n-1)) = \theta(1/n^\wedge 2)$, this forces us to cover all the elements.
- Giving a $m \log (p\ n^\wedge 2) = O(m \log (n))$ approximation. Matches the set cover lower bound.

If We Don't Have Exact VIQs



Discussions and Open Problems

- Interesting model to study various hidden structures.
- Linking value injection query model to real-world problems.
- Finding non-path based methods for social networks (and probabilistic networks)
- Reducing Query Size